ADVANCED MASTERS IN STRUCTURAL ANALYSIS
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Master's Thesis

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Numerical Model and Seismic Structural Analysis of the Lamberti Tower

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DECLARATION

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ABSTRACT

Masonry towers are relevant typology buildings in the Italian built heritage, due to their widespread in the territory and the important historic value that they have. The history shows emblematic examples of the consequences of their structural vulnerability. In fact only a few of these structures survived until today and the majority of the ancient high towers in Italy are no longer present. The high structural risk of high masonry tower justified detailed studies and carefully planned remedial measures.

The present Master Thesis project focuses on the Lamberti Tower, a medieval masonry tower with a height of 84 m located in Verona, northern Italy. The work starts by describing the research that was already done on the tower and that was made available. Then, a numerical model of the structure was built to be able to simulate its behavior.

Two different models were prepared with 3D elements based on the Finite Element Method. In the specific case of the towers, the simplicity of the structure allows to create models that replicate well their thickness and the materials present without using too many elements. For this reason, a simple model composed by 3D composite beams was made, which allows future analysis using time history analysis. The second model is more detailed and was defined using 3D solid elements, which make it mode adequate for static analysis.

In order to update the Finite Element (FE) models dynamic identification has been carried out on May 29th, 2013. The aim of the acquisitions was to define the modal parameters (natural frequencies and mode shapes) of the building. The numerical results, obtained from the structural eigenvalue analysis of the solid model, were compared with the experimental values and it was observed that the first adopted physical properties were not acceptable, possibly due to the boundary conditions. For this reason a modal updating, for the 3D solid mesh, was performed to validate the models and the modal assurance criterion (MAC) was calculated. The model built showed a behavior similar to the real tower, but a deeper investigation is recommended, especially for what concern the material parameters of the different types of masonry that compose the structure and the boundary conditions.

At the end of this work the 3D beam model and 3D the solid model were used for a subsequent non-linear static (pushover) analysis for a first seismic assessment purpose. The strain distribution found showed that when the tower reaches considerable displacements, a failure mechanism with rotation at the base of the tower was clearly defined.

RESUMO

Torres são edifícios de alvenaria tipologia relevantes no património edificado italiano, devido à sua ampla no território e do valor histórico importante que eles tenham. A história mostra exemplos emblemáticos das conseqüências de sua vulnerabilidade estrutural. De facto, apenas algumas destas estruturas e sobreviveram até hoje a maioria das torres altas antigos em Itália não estão mais presentes. O alto risco estrutural de alta torre de alvenaria justifica estudos detalhados e cuidadosamente planejado de medidas correctivas.

A presente Dissertação de Mestrado projecto centra-se na Torre Lamberti, uma torre medieval de alvenaria com uma altura de 84 m localizado em Verona, norte da Itália. O trabalho começa por descrever a pesquisa que já foi realizado na torre e que foi disponibilizado. Em seguida, um modelo numérico de a estrutura foi construída para ser capaz de simular o seu comportamento.

Dois modelos diferentes foram elaboradas com elementos 3D baseados no Método dos Elementos Finitos. No caso específico, as torres, a simplicidade da estrutura permite que a criação de modelos que replicam bem a sua espessura e os materiais presentes, sem o uso de muitos elementos. Por esta razão, um modelo simples, composta por vigas mistas 3D foi efectuada, o que permite uma futura análise utilizando a análise de histórico de tempo. O segundo modelo é mais detalhado e foi definida utilizando elementos sólidos 3D, que o tornam adequado para a análise do modo estático.

Para atualizar o Elementos Finitos (FE) modelos de identificação dinâmica foi realizada em 29 de maio de 2013. O objetivo das aquisições foi para definir os parâmetros modais (freqüências naturais e modos de vibração) do edifício. Os resultados numéricos obtidos a partir da análise do modelo sólido eigenvalor estrutural, foram comparados com os valores experimentais e observou-se que as primeiras propriedades físicas adoptadas não foram aceitáveis, possivelmente devido às condições de contorno. Por esta razão, foi realizado um modal de atualização, para a malha 3D sólido, para validar os modelos ea garantia modal critério (MAC) foi calculado. O modelo construído apresentou um comportamento semelhante ao real torre, mas uma investigação mais profunda é recomendada, especialmente para os que dizem respeito aos parâmetros materiais dos diferentes tipos de alvenaria que compõem a estrutura e as condições de contorno.

No final deste trabalho, o modelo 3D de feixe e do modelo sólido em 3D foram utilizados para uma análise não linear estático (influenciável) subsequente para um primeiro objectivo sísmica. A distribuição da tensão encontrada mostraram que quando a torre atinge deslocamentos consideráveis, um mecanismo de falha com rotação na base da torre foi claramente definida.

SOMMARIO

Titolo: MODELLO NUMERICO E ANALISI SISMICA DELLA TORRE DEI LAMBERTI

Le torri in muratura rappresentano una tipologia strutturale importante nel patrimonio architettonico italiano, per l'ampia diffusione sul territorio e per il valore storico che rivestono. In passato vi sono stati esempi emblematici della loro vulnerabilità strutturale. Solo alcune di queste costruzioni, infatti, sono sopravvissute fino ad oggi e la maggior parte delle prominenti torri medievali sono scomparse. L'elevato rischio strutturale delle torri giustifica l'elevato numero di studi effettuati e delle avanzate misure d'intervento e di consolidamento pianificate.

Il presente progetto di tesi di Master si focalizza sulla Torre dei Lamberti, una torre medievale in muratura alta 84 m e ubicata nel centro storico della città di Verona, in Italia settentrionale. Nella parte iniziale dell'elaborato vi è la descrizione di una relazione precedente realizzata e resa disponibile da uno studio ingegneristico italiano. Dopo una descrizione dello stato dell'arte della Torre è stato definito un modello numerico per simulare il comportamento della struttura.

Due differenti modelli sono stati elaborati con elementi 3D basati sul metodo agli elementi finiti. Nello specifico caso delle torri, la relativa semplicità strutturale permette di creare modelli che replicano in modo efficace il loro spessore e le loro caratteristiche dei materiali senza l'uso di troppi elementi. Per questa ragione, un semplice modello costituito da elementi beam 3D è stato realizzato, il quale permette di eseguire future analisi dinamiche non lineari. Il secondo modello, più dettagliato, è stato definito usando elementi solidi 3D, il che lo rende più adeguato per delle analisi strutturali statiche.

Con lo scopo di affinare i vari modelli agli elementi finiti (FE) ottenuti, il giorno 29 Maggio 2013 sono state effettuate della prove di identificazione dinamica sulla Torre dei Lamberti. Lo scopo delle acquisizioni è stato quello di definire i parametri modali della struttura (frequenze proprie e funzioni di forma). I risultati numerici, ottenuti dall'analisi agli autovalori del modello con elementi brick, sono stati confrontati con i valori sperimentali ed è stato possibile osservare che le proprietà fisiche inizialmente adottate non erano accettabili, probabilmente per le condizioni al contorno considerate. Per questa ragione, è stata eseguita una calibrazione sulla mesh del modello 3D con elementi brick per validare i differenti modelli, ed è stato infine calcolato il "modal assurance criterion" (MAC). Il modello realizzato ha mostrato un comportamento simile a quello reale della torre, anche se sarebbe consigliata un'indagine più approfondita, soprattutto per quanto concerne i parametri dei materiali che costituiscono i diversi tipi di muratura della struttura e pure per le condizioni al contorno della torre.

Nella parte conclusiva di questo elaborato il modello beam 3D e il modello con elementi solidi 3D sono stati utilizzati per un'analisi statica non lineare (pushover) al fine di ottenere una prima valutazione simica.

La distribuzione delle deformazioni ottenuta ha evidenziato che, quando la struttura raggiunge spostamenti considerevoli, è chiaramente individuato un meccanismo di rottura con una rotazione alla base della torre.

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1. INTRODUCTION

The analysis of historical masonry constructions is a very complex procedure; it presents often issues, as lack of knowledge of: geometry of the elements, mechanical properties of the material, different historical construction phases, damages of materials and elements. The conservation of the historical architectural heritage by a structural analysis is a theme linked with multiple fields and is articulated in different phases with the goal to reach an appropriate level of knowledge of the structure.

The most important activities in the studying of historical buildings are: historical research of the principal construction phases and of the exceptional events linked with the structure; inspection and characterization of the current status of the construction; monitoring of the building; modeling and analysis of the structure (Roca, 2005).

This Master Thesis project focuses on a medieval masonry tower located in Verona (Italy). Many analyses and researches have been carried out in the last century about masonry towers, because a lot of them had many damages after earthquake and because some towers in a short time, or even suddenly, collapsed. A key aspect in the behavior of ancient towers is that the collapse process usually excludes the possibility of ductile behavior. There are, in fact, hardly possibilities of internal force redistributions between the different critical sections, and failure of a single section is usually sufficient to provoke the entire collapse of the structure. This intrinsic feature leads to a high structural risk in tall masonry towers, because increasing height means large vertical loads and high compressive stresses at the base. Therefore, it seems easy to accept that masonry towers should possess a higher safety margin than the values normally found for other historical structures.

2. MASONRY TOWERS

Masonry towers are relevant typology buildings in the Italian built heritage, due to their widespread in the territory and the important historic value that they have. Towers are structures characterized by the prevailing development of the height when compared to the other dimensions.

Ancient builders tried to challenge the structural stability and the nature itself by making towers, the results of the need of cities or families to show their power. It is possible to observe a very unique example of this "competition" to build taller structures in the city of San Gimignano in Toscana. In the fourteen century the city had over seventy towers, built by the many wealthy families in the town as a way to show their wealth and power. Only striking fourteen towers, of various heights, have withstood wars and time, and they continue to define the city making its unique skyline an international symbol. It is possible to understand from the few remaining towers of this city that ancient builders made rather vulnerable structures. In fact, it is striking that most of the ancient high towers in Italy, e.g. Pavia and Bologna, are no longer present, due to collapses by exceptional events (e.g. earthquakes and lightening), sustained loading and deterioration, and even demolitions (often by precaution and concern of eminent collapse).

For these buildings we have emblematic examples of the consequences of the structural vulnerability, such as the collapse of the Campanile of San Marco in Venice, which took place in 1902, and the collapse of "Torre Civica" of Pavia in 1989 (Binda et al., 1997). In addition, the observation of the damages caused by earthquakes (Friuli 1976, Emilia Romagna 1996, Umbria-Marche 1997, Piedemonte 2000, Molise and Puglia 2002, Salo 2004, Abruzzo 2008, Modena 2012) allows better understand of the dynamic behavior of masonry towers.

Brick-masonry load bearing elements subjected to high stresses, such as towers, frequently exhibit typical mechanical deterioration phenomena like formation of vertical or sub-vertical thin and diffused cracks, together with local detachment of the outer leaf in multiple leaf walls. Such a particular crack pattern is often not attributable to common causes of damage like seismic events, foundations settlements, excessive external loading or chemical, physical and mechanical degradation of the basic material. On the contrary, this is often due to the prevalent effect of the dead load and to the associated time dependent phenomena (Modena et Valluzzi, 2003).

In order to understand a structure different information must be obtained, such as: the geometric survey, the survey of the damage, the survey of deterioration of the materials, the identification of the mechanical properties of the materials, and the knowledge on the structural behavior. The need of conservation of historical monuments led to the development of in situ non-destructive or minor-destructive techniques. The tests carried out in the laboratory are usually limited to determine the chemical and physical characteristics of very small material samples, because it is usually not possible to remove large samples of masonry for mechanical tests. Sometimes, cores can be obtained and mechanical characteristics can be obtained for the masonry components. Also the global characteristics of existing structures can be obtained with dynamic identification tests.

2.1 THE COLLAPSE OF THE CIVIC TOWER OF PAVIA

An important case in the investigation of the masonry towers is given by the Civic Tower of Pavia and its collapse (Binda et al., 1990; Binda et al., 2007). The tower was made of brick masonry, originally, from the eleventh century, and suddenly collapsed on 17 March 1988. This event allowed making mechanical tests in the laboratory using parts of the masonry recovered from the rubble. The results of these tests, impossible to carry out in existing buildings, helped to identify the causes of the collapse and made possible to formulate a theory about the behavior of masonry subjected to heavy sustained loadings.

Initially, several hypotheses were made about the causes of the sudden failure, from soil settlements to the presence of a bomb, from vibrations caused by car and air traffic. The tower was about 60 m high with a square base measuring 12.3 m and was located close to the north-west corner of the Pavia Cathedral. Each of the four faces of the tower was divided horizontally into six levels. The first four levels were divided into five parts by four pilasters. The fifth level terminated in a cornice. A large mullioned window with two apertures opened out on each side of the sixteenth-century belfry. Inside the tower, two timber floors existed at a height of approximately 11 and 23 m. Between 1583 and 1598 the granite belfry weighing 3000 tons, designed by the famous architect Pellegrino Tibaldi, was set on top of the tower. A staircase built into the wall ran along all four walls from the southwest corner up to the belfry.

The walls have been built according to the techniques usually normal in the medieval time for towers: two external brick leaves ranging from 120 to 400 mm with an average of 150 mm, with a core consisting of irregular courses of large pebbles of brick and stones alternated with mortar, constituting a sort of conglomerate. The walls of the second building phase were characterized by a much more irregular filling and by thinner external leaves. The staircase was covered by a small barrel vault apparently made of conglomerate. The first investigations, in situ and in the laboratory, were about the characteristics of the soil in the foundation, as towers have very high stresses and tend to rotate due to the settlements and lateral connections to other building parts. The values obtained have been used to calculate the load bearing capacity of the soil. The loadbearing capacity of the soil was 1.16 N/mm^2 and the safety factor was 2.4 and 4.0, sufficient to guarantee the stability of the foundation. The ultimate differential settlement was estimated at 5 mm and was considered irrelevant to affect the static conditions of the structure. For these reasons the cause of the collapse was not linked with the foundations of the tower. The physical, chemical and mechanical tests on the material showed a superficial degradation depth of 80-100 mm. Mechanical tests have been made on small specimens, with bricks and mortar, and in large masonry specimens (100 x 100 x 200) mm^3 that have withstood after the collapse. The compression tests on cubical mortar specimens (with a side of 2.7 and 3.5 mm) gave a compressive strength of 6.5 N/mm^2 and an elastic modulus of 905 N/mm^2. From the compressive tests on bricks (cubic specimens with side 40-50 mm) a compressive strength of 13.37 N/mm^2 and an elastic modulus of 1973 N/mm^2 were obtained. Other tests have been carried out on masonry prisms: monotonic compression tests to failure, loading and unloading cyclic tests, and fatigue tests. The values obtained by the different specimens exhibited a large scatter.

Also sustained creep load tests have been carried out to evaluate the influence of the time of application of the load on the masonry. For the creep load test, most prisms were tested under load control up to 1.0 – 1.5 N/mm^2. The stress was increased in steps of 0.14 N/mm^2, at intervals of at least 15 min. The increase of strain was on average 1.6×10^{-3} for each 15 min interval at constant load. At higher stresses, close to the ultimate strength of the material, the time-dependent effects of the constant load evolved more rapidly. No further load increases were made until the increase in strain stopped. At the last step the strain rate continued to increase rapidly until failure occurred suddenly after a period of time between 10 min and 2 or 3 hours. It has been assumed that the time needed to reach collapse is a function of the ratio between the load applied and the maximum load that the specimen is able to withstand. The development of the cracks has been interesting: until the condition close to the collapse only a small number of vertical cracks appeared on the bricks and on the mortar joints. These cracks started to appear at loads over approximately 70% of the ultimate compression strength: similar cracks, vertical and very thin, are visible on the external façade of the tower in a picture of 1968. At the end of this investigation it a FEM model about the first 22 m of the tower (the first and the second phase of the construction) has been elaborated to evaluate the distribution of the stresses due to the self-weight and the wind load. This model showed that some areas at the base of the tower present compression stresses values very close to the maximum load of collapse obtained in the laboratory; in particular values between 1.6 and 2.0 N/mm^2, range where the cracks started to develop. At the end of the investigation it has been found that the reason of the collapse was the self-weight of the tower, and, for the first time, the hypothesis of a collapse of an ancient masonry due to the long-term behavior of the material has been formulated. Afterwards a creep mathematical model based on this hypothesis for the collapse of masonry has been formulated (Anzani et al., 2000).

In conclusion, the failure of the Civic Tower of Pavia allowed paying attention on the creep phenomenon, which is of interest to structures under constant loads close to the ultimate load of the material. This condition, in fact, causes the development of widespread thin and small cracks on the base of the structure, which produce, in a long period, failure. Failure seems to happen suddenly, without apparent warning (such as large cracks or spalling), even in the proximity of collapse.

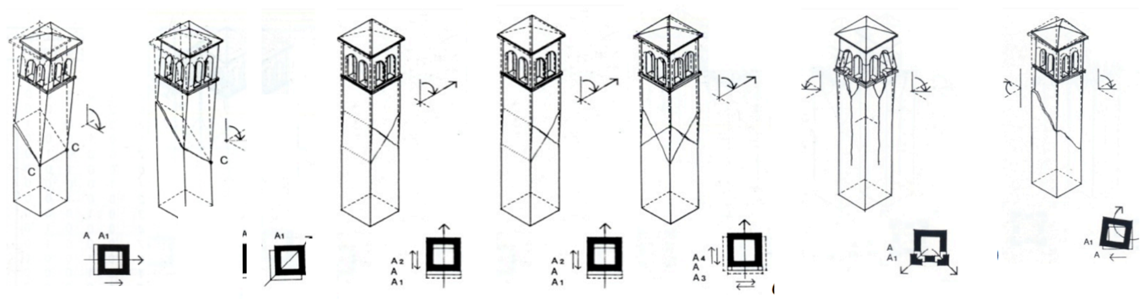
2.2 ANALYSIS OF CRACKS AND DAMAGES

The analysis of cracks and other damage allows the formulation of a first hypothesis about the structural condition. If this is not easy to understand there are complementary test-inspections that can be done. The survey of the cracks may be difficult in the case of the towers due to the accessibility. Some causes of recurring damage involving specific crack patterns have been identified for masonry towers: the crushing of the most compressed parts caused by the self-weight of the structure and the inclination of the main axis from the vertical line due to differential subsidence of the soil (Binda et al., 1997). The damage due to an earthquake is determined by various parameters such as: the number and arrangement of the openings along the shaft of the tower, the connection with adjacent buildings, and the presence of materials with different mechanical characteristics. In the particular case of bell towers also the action of the bells could lead to the formation of damages, especially if the resonance phenomenon appears (Mammino et al., 2000; Bennati et al., 2005b).

The towers subjected to problems of crushing may present damages in the middle and lower zones. In this category, for instance, there are the bell tower of the Cathedral of Monza (Modena et al., 2002) and the bell tower of Santa Giustina in Padua (Valluzzi et al., 2005). In towers subjected to problems of inclination, crack patterns have been observed, characterized by damages in the lower part of the under slope due to the larger compressive stresses (Binda et al., 1997). Another cause of damage of the towers has been identified in the use of materials with different mechanical properties. It is possible to say that the presence of two materials with different deformability determines a strain distribution in inverse ratio of their respective elastic modulus. For example this phenomenon has been found in the bell tower of the Cathedral of Avellino (Mastrodicasa, 1993) in which the lower part of the wall is made by an external layer of 0.4 m of limestone while in the internal part there is a layer of 1.2 m made by tuff blocks. The tuff has an elastic modulus much lower than the limestone; this caused the formation of damages such as to cause the separation of the contact surfaces that behave independently.

Damages and collapse due to seismic action are very different from the above and are activated by specific characteristics of each structure. In the dynamic response of an historical monument in masonry is very important the identification of the kinematisms, which could create the collapse of the structure. An extensive survey of the damage to bell towers was executed as a result of the earthquakes of Friuli in

May 1976 and September 1976 (Doglioni et al., 1994). Mechanisms relating to 60 quadrangular towers have been considered. The damage analysis led to the consideration of two distinct macro-elements: the tower and belfry. The mechanisms related to the element tower have been linked to the division in two typology classes: the relation with adjacent buildings and the distribution of the openings on all four elevations. Six mechanisms are identified for the element tower: 1) out of plane rotation of the upper part of the tower due to actions out of plane (Figure 1a); 2) translation of the upper part of the tower, followed by the rotation of the entire tower; about the four sides of the tower two of them are failing in plane and two of them are failing out-of-plane (Figure 1b); 3) out-of-plane rotation in a hinge due of damages that create inclined cracks; 4) out-of-plane rotation of one or more inclined crack (Figure 1c); 5) rotation of the upper part of the tower resulting from the combination of two rotations around a vertical axis and a horizontal axis (Figure 1d); 6) translation of the upper part because of horizontal damage (Figure1e)

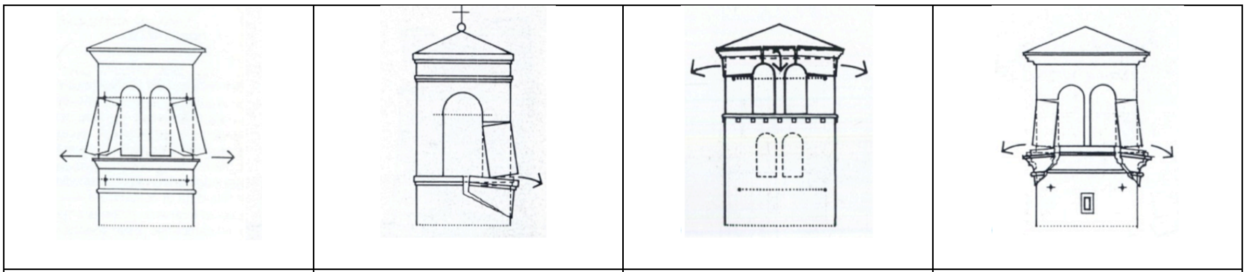


(a) (b) (c) (d) (e)

Figure 1 – Mechanisms for towers: a) kinematism 1; b) kinematism 2; c) kinematism 4; d) kinematism 5; e) kinematism 6 (Doglioni, 1994).

Mechanism 2 has been detected in a larger number of cases, particularly associated with isolated bell towers, which have no contact with other parts of the building. Mechanism 5 of rotation occurs also in a large number of cases, when the tower is connected with adjacent buildings.

For the macro-element formed by the belfry, three mechanisms have been identified: 1) translation or rotation-translation of the pillars (Figure 2a); 2) translation or rotation-translation of the pillars with shearing-off of the top rail (Figure 2b); 3) roto-translation out-of-plane of masonry at the base of the pillars (Figure 2c). It has been found that the damages could develop mainly at the ends of the pillars and at the middle of the beam. Moreover it has been observed that this type of damage and collapse is determined more by the characteristics of the masonry than the typology of the belfry.



(a)	(b)	(c)	(d)

Figure 2 – Mechanisms of bell towers considering the belfry: a) kinematism 1, b) kinematism 2, c) e d) kinematism 3 (Doglioni at al., 1994).

The type of seismic damage in isolated towers was detected in the assessment of the damage caused by the earthquake of 1997 Umbria-Marche (Marchetti et al., 2004). The inspection has highlighted the formation of vertical cracks in correspondence of the openings, such as to divide the structure into parts corresponding to the corners of the tower. This mechanism is visible in the tower of Triponzo in Cerreto di Spoleto (Figure 3), which was partially collapsed during the earthquake of 1979. The subsequent intervention has been aimed to restore the missing parts and to build a curb on the top During the earthquake of 1997 rebuilt parts collapsed again in the same mechanism of damage.

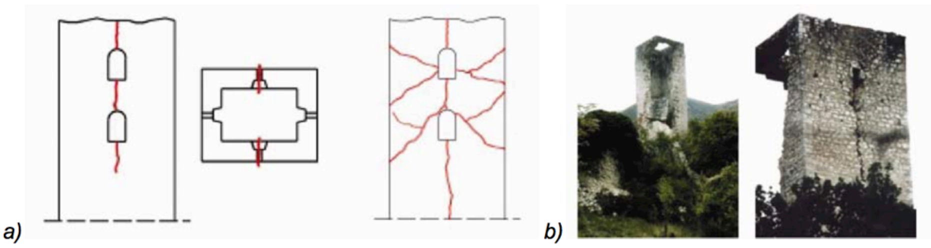


Figure 3 – Seismic damages of isolated towers after the earthquake in 1997: a) different pattern of damage; b) example of Tiponzo Tower in Cerreto di Spoleto (Marchetti et al., 2004).

2.3 SAFETY ASSESSMENT METHODS

Investigations techniques are important to reach a good judgment about the conservation and the safety condition of the structure, considering the dead load, wind or seismic actions. To understand better an old building it is also necessary to adopt models that can replicate the behavior of the structure. There are different levels to verify the safety condition of a building suggest in the Italian Code (Linee Guida, 2010). A first level to evaluate the condition of the structure is the application of simplified models which depend on the typology of the structure. A second level is the analytic verification of the local mechanisms, again divided on their typology. Finally, a third level is the evaluation of the safety condition with mathematical advanced models.

2.3.1 SIMPLIFIED VERIFICATIONS

There are simple verifications that have been proposed (Binda et al., 1997) to find very easily dangerous situations in a structure, using only the principal dimensions and morphological characteristics. These calculations allow finding the stress at the base and the flexural failure, which can be compared with other cases. The work carried out in the Politecnico di Milano for towers (Binda et al., 1997) shows dangerous situations regarding the stresses at the base. The values have been obtained considering for brick masonry a specific weight of 18 kN/m^3 and for stone masonry a specific weight of 20 kN/m^3. The values for different towers have been compared with the average and maximum values on the section of the base of the Civic Tower of Pavia and of the bell tower of Duomo di Monza. The buckling has been calculated considering two limit conditions: the first one, optimistically, considers the masonry walls perfectly clamped and the entire section active; the second one, conservatively, considers the walls not clamped. The real buckling behavior will be between these two described limits. For this simple methodology of analysis, an elastic modulus of 2000 N/mm^2 for the brick masonry and an elastic modulus of 4000 N/mm^2 for stone masonry were used. These simple verifications are useful to indicate particularly unsafely cases.

About the seismic verification, the Italian Code "Linee Guida (2010)" indicates the need to verify the flexural failure, which considers the structure like a cantilever with the vertical dead load and a horizontal force of the earthquake. For rectangular hollow sections, which are the most common for towers, a formula to calculate the resistant bending moment M_u of the section has been proposed

$$\boldsymbol{M_u = \frac{\sigma_0 A}{2}\left(b - \frac{\sigma_0 A}{0.85 \cdot a \cdot f_d}\right)} \qquad (1)$$

where:

a is the perpendicular side to the direction of the earthquake
b is the parallel side to the direction of the earthquake
A is the total area of the section
$\boldsymbol{\sigma_0}$ is the normal stress on the section, =W/A where W is the weight of the structure above the section
$\boldsymbol{f_d}$ is the compression strength of the masonry

The total applied bending moment is calculated considering a system of forces distributed along the height of the structure.

2.3.2 ANALYTICAL VERIFICATIONS

In the last years several earthquakes caused strong damages in historical constructions. The inspection of the collapses has allowed identifying specific damage mechanisms in towers and bell-towers, as discussed above. The safety verification method with limit analysis of the equilibrium needs the identification of the kinematisms that the structure can develop. Each structure is a single case and the mechanisms of the “Linee Guida, 2010” (Italian Code) to be applied require experience and a critical review.

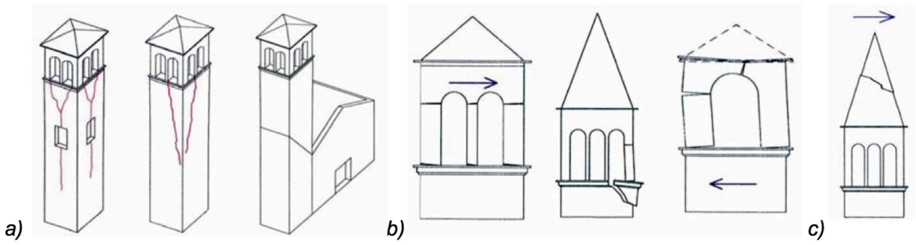


Figure 4 – Collapse mechanisms for bell towers (Linee Guida, 2010): a) tower, b) belfry, c) roof

In the verification of the stability the masonry the material is considered without tensile strength and with infinite compression strength (Heyman, 1992). These considerations have been made during the inspection of the damages on medieval tower in Cambridge, with diagonal inclination on the parallel sides of the slope (Figure 5a). By a mathematical analysis it has been possible to describe the crack pattern and to calculate the limit slope for the stability of the construction. Considering a parallelepiped with height a and base b, inclined with an angle α (Figure 5b), the results are that in uplifting part the stress is $\boldsymbol{\sigma = 0}$. The value of α to reach this condition is:

$$\boldsymbol{tan\alpha = \frac{1}{3} \cdot \frac{b}{a}} \tag{2}$$

Considering now a parallelepiped with generic inclination (Figure 5c), it is possible to calculate a from the previous equation: the distance from top such that at the left corner there is zero stress. In each section in a generic distance X from the top it is possible to consider the reduced section Y, where the reaction force will be applied at 1/3 of Y.

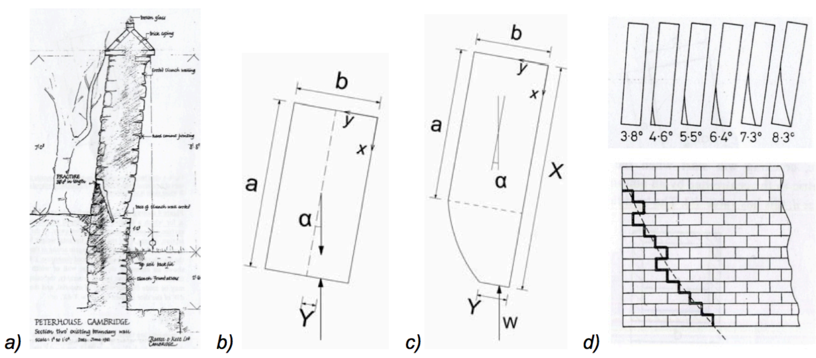


Figure 5 – Leaning towers: a) Medieval Tower of Peterhouse, Cambridge; b) inclined parallelepiped; c) parallelepiped with generic inclination; d) crack development (Heyman, 1992).

From the results of the equilibrium equations in the kinematism of the small part with height dX it is possible to obtain the parametric equations that described the curve $\boldsymbol{\sigma = 0}$ (the line of the crack in the masonry) that are expressed in the variables x=X/a, y=Y/b (Figure 5d).

$$\boldsymbol{x = 1 + \sqrt{\pi} \cdot e^{\frac{1}{4}} \left(erf\,\frac{1}{2} - erf(t) \right)} \tag{3}$$

$$\boldsymbol{y = e^{\frac{1}{4}} \cdot 2t \cdot e^{-t^2}} \tag{4}$$

The ultimate inclination is reached when the crack line passes through all the depth of the tower, which is the condition y=0 and provides:

$$\boldsymbol{tan\alpha = \frac{0.7282}{\frac{H}{b}}} \tag{5}$$

where H is the full height of the tower. This solution is valid for full sections, while usually real cases are characterized by hollow sections. For these hollow sections there is an approximate solution where the crack line is considered linear and the section thin: b=c, where b is the external dimension of the base and c the internal dimension. The approximate solution provides the maximum angle of inclination:

$$\boldsymbol{\alpha = 48(b/H)} \tag{6}$$

Comparing the angles of maximum inclination it is possible to observe that, with the same dimensions, the approximate solution for full section provides a critical angle lower than the one for hollow sections (Refer as Table 1). The angle when the first crack appears at the base of the tower has been calculated also: $\boldsymbol{tan^{-1}(\frac{1}{3} \cdot \frac{b}{H})}$ for the full section; $\boldsymbol{tan^{-1}(\frac{2}{3} \cdot \frac{b}{H})}$ for the hollow section. The angle of inclination that corresponds at 24(b/H) is the ultimate safety value; beyond this value stability is no longer ensured.

	H/b						
	3	4	5	6	8	10	12
OVERTURN							
Solid section	13.4	10.1	8.1	6.8	5.1	4.1	3.4
Hollow section	15.7	11.9	9.6	8.0	6.0	4.8	4.0
FIRST CRACK							
Solid section	6.3	4.8	3.8	3.2	2.4	1.9	1.6
Hollow section	12.5	9.5	7.6	6.3	4.8	3.8	3.2

Table 1 – Angles of maximum inclination for different values of ratio H/b (Heyman, 1992)

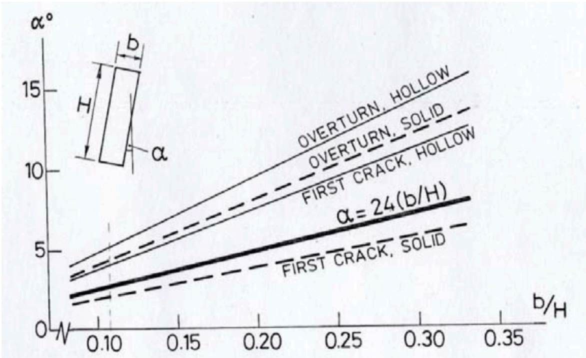


Figure 6 – Leaning towers: critical angle of inclination for different values of the ratio base/height (Heyman. 1992).

The solution that has been found is valid for inclinations parallel to one of the sides of the rectangular section. For instance to evaluate the safety condition of the bell tower of San Benedetto in Ferrara the analytical formulation has been generalized, for any inclination of the axis of the tower with respect to the vertical axis and the axes parallel to the section of the base (Laudiero et al., 2003).

2.3.3 MATHEMATICAL ADVANCED MODELS

Mathematical models of a structure could be developed using different methods considering the aim of the analysis and the level of knowledge of a building and its characteristics. The computational burden of data processing is currently easy thanks the use of computers; for this reason today the finite element method using 2D and 3D elements is very used. In the specific case of the towers the simplicity of the structure allows to create models that replicate well their thickness and the materials present without using too many elements. Masonry has a non-linear behavior in tension and compression and is also anisotropic. However, in many cases, a linear isotropic model is used for the material, due to the computational efforts required by non-linear material models, because is not easy to know the properties of a historical masonry in detail and also because many users do not have sufficient skills to adequately used non-linear analysis.

The dimension of the height of a tower is bigger that other dimensions, this allows also to use 1D models (Cerioni at al., 1996; Riva et al., 1998). One possibility is to decompose the structure in beam elements with linear-elastic behavior and to concentrate the non-linearity in the nodes. For instance the model of the Tower of Asinelli in Bologna (Riva et al., 1998) is made with beam elements with elastic behavior (E=5000 N/mm^2; ρ=1800 kg/m^3) linked with truss elements in the nodes. The truss elements have deformability only along their axis and different values of ultimate tensile and compressive strength.

Also in the frame of 3D models, 1D elements have been used (Aoki et al., 2006; Ceravolo et al., 1999). These types of models have the advantage to show a clear interpretation of the dynamic behavior of the tower using a very simple model. The FEM method can be used for 2D models that represent a façade of

a building (Bartoli et al., 2006; Cerioni et al., 1996; Thomas et al. 1996; Valluzzi et al., 2006). These models have been used to evaluate the in plane behavior of the masonry under different load conditions.
Still, the most used method for towers seems to be FEM 3D with solid volume elements "brick" (for instance: Abruzzese et al., 2005; Barsotti et al., 2007; Balduzzi et al., 2006; Benedetti et al., 2007; Carpinteri et al., 2005; Cerioni et al., 1996; D'Ambrisi et al., 2004; Gentile et al., 2007; Ivorra et al., 2006; Lionello et al., 2005; Pavese et al., 1995; Rebelo et al., 2007; Rossi et al., 1992). These models allow a good geometry representation and the identification of the materials. In some cases, the models show also the soil, with the goal of understanding the interaction between the ground and the foundations of the structure (Abruzzese et al., 2005; Beconcini et al., 2006). If there are other buildings near the tower, it is possible to make the constraints modeling these parts of the structure (Decanini et al., 1997; Lionello et al., 2005). At the beginning of the modeling it is possible to assign the characteristics of the materials. It is also necessary to assign many parameters, in particular for the non-linear and anisotropic behavior, that often are not easy to interpret. Another strategy for towers modeling is the utilization of 3D FE models with shell elements (Reale et al., 2001; Ramos et al., 2007) that have the advantage to be a lower burden from a computational point of view.
The numerical models can be used to verify the safety of the structure under an assigned load, but also to interpret the results of experimental tests and dynamic tests. The most common analyses are linked with the dead load, wind action and seismic action. Two verifications are particularly relevant: the inclination of the axis of the tower and its increase (Carpinteri et al., 2005); the non-linear time dependent behavior in compression, which is defined as creep (Papa et al., 2001). The safety verification of the wind action is usually made considering this action like a pressure statically applied (Carpinteri et al., 2005; Modena et al., 2002; Rossi et al., 1992; Valluzzi et al., 2003), neglecting possible dynamic effects. Further safety verification of the structure is made implementing the stresses of the thermal changes (Modena at al., 2002) that in the case of towers can be considerable.
The seismic verification is made using 4 methods: linear static analysis, non-linear analysis, dynamic linear analysis and dynamic non-linear analysis. From the code in Italy, "Linee Guida (2010)", the static non-linear verification needs the application of two forces: one proportional to the weight and one proportional to the first vibration mode. In the case of towers and bell towers, it is interesting to consider the distribution of the forces proportional to the vibrations modes higher than the first one (Ceroni et al., 2007). The dynamic analysis provides the application of a time history to the finite elements model where in non-linear analysis the materials have non-linear characteristics (Abruzzese et al., 2004; Azorin et al., 2006; Ivorra et al., 2006). This makes this analysis particularly burden from the computational point of view; that is why is easier using simplified models also for the towers (Cerioni et al., 1996; Riva et al., 1998).

2.4 REPAIRING AND STRENGTHENING TECHNIQUES

From the identification of the structural specific problems it is possible to choose the intervention for repairing and strengthening. Each strengthening techniques is designed for a single damage or multiple damages.

Towers are often subjected to settlements of the soil under the foundations. In the past deep underpinning with micropiles has been used with the aim of rigidly securing the structure to the substrates of the soil that have better mechanical characteristics (Mastrodicasa, 1993). Today the intervention techniques, that could be less invasive, can interest the substructure and the foundations. For instance the strengthening operation of the bell tower Santa Maria Gloriosa dei Frari in Venice (Lionello, 2008) where the soil fracturing technique was used, consisting of high pressure injection of cement mortar to increase the mechanical characteristics of the clay soil foundations.

Regarding the structure there are two common techniques used to strength masonry and towers: injection of lime based grouts (Ceravolo et al., 1999; Modena et al., 2002; Valluzzi et al., 2005) and the reparation with the "scuci-cuci" technique (local repairs by reconstruction) of the most damaged parts (Indirli, 2000; Modena et al., 2002; Valluzzi et al., 2005; Vegas Lopez-Manzanares et al., 2005).

2.4.1 TIE RODS

Another traditional strengthening technique is the utilization of steel tie rods. This technique improves the connection between the walls and also increases the confinement of the masonry. In slender structures like towers the connection between adjacent walls is very important, as there are usually damages in the corners that induce a non-box behavior of the construction. This technique has been used in many interventions, for instance the bell tower of the Cathedral of Avellino (Mastrodicasa, 1993), tower of Fraccaro in Pavia (Ballio, 1993), the bell tower of Maria SS. Annunziata in Roccaverano (Ceravolo et al., 1999). The tie rods have been placed in different levels of the structure in order to confine the walls.

For instance an interesting case is the intervention with a series of stainless steel reinforcing rings that have been planned to be applied in different horizontal sections of the Monza Cathedral bell-tower (Modena et al., 2002). These reinforcing rings have been properly designed depending on the particular local conditions detected on the structure. There are different types such as "A" types are composed by two internal rods (30 mm in diameter) anchored to stainless steel plates which will be possibly included in proper recesses successively closed by masonry. This is complemented by the general solution for the anchoring of the ties, the "A2" type, characterized by the local strengthening of the masonry by means of a series of steel bars diffused along the connection zone. The vicinity of the church at the first level led to the "A1" type solutions, performed by the transversal connection of the anchoring ties. Because of the presence of a vault at the height of 11 m from the bell-tower a C-shape metallic components and tie rods, types “B”, anchored by plates not visible from outside were also applied. The only reinforcing ring external to the structure, type “C”, has been placed at the cornice level composed by two tie rods (36 mm in diameter). It has been decided also to apply a further reinforcing ring at the foundation level, type “F”, executed by four reinforced concrete beams connected by a series of post-tensioned bars inserted in cored holes.

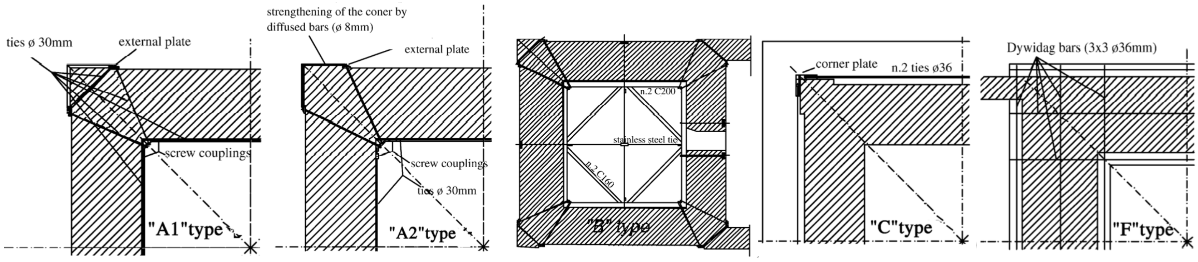


Figure 7 – Different types of reinforcing rings in the bell-tower of Duomo di Monza: a) A1 type; b) A2 type; c) B type; d) C type; e) F type

Another interesting case is the strengthening intervention of the Civic Tower of Vicenza (Valluzzi et al., 2003). Also in this case the stainless steel reinforcing rings designed have been of three different kinds. Type “A” has been used to reinforce the octagonal tip of the tower. Type “B” and “C” have been used along the height of the tower, at seven different levels. All the reinforcing rings have been specifically designed on the basis of the particular conditions and geometry of the structure. The original design with an anchoring plate have been abandoned and substituted with a stainless steel cross in order to reproduce the historical element found on a pillar of the belfry (Figure 8a and b). Type “C” has been placed at the base of the tower, composed by two rods for each wall (Figure 8c). Moreover, small diameter (6 mm) stainless steel reinforced bars were placed in the mortar joints at the same height of the reinforcing rings, in order to assure a confining action at the corners of the walls.

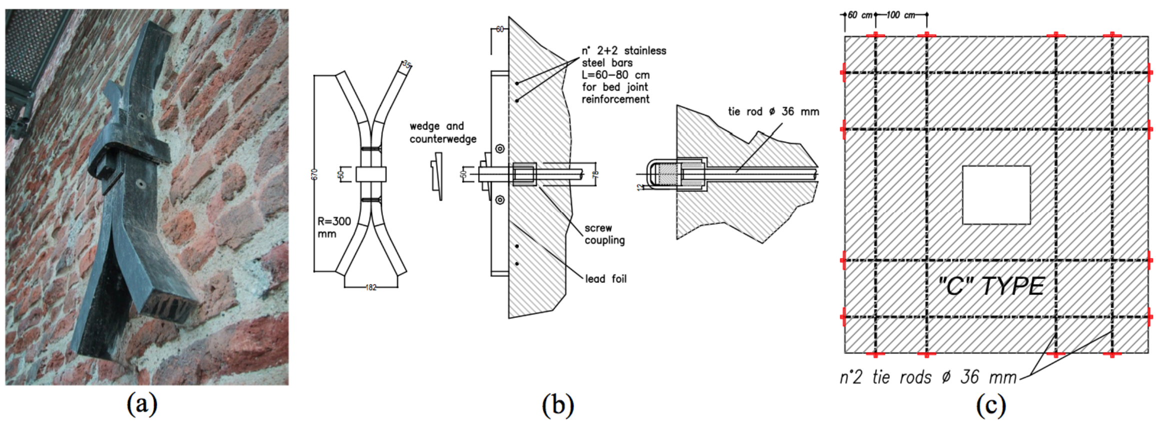


Figure 8 – Reinforcing rings in the Civic Tower of Vicenza: a) external view; b) scheme; c) new anchor element and scheme of the “C” type of reinforcing ring.

In both the interventions of the bell-tower of Duomo di Monza and of the Civic Tower of Vicenza reinforced repointing has been used, in the part affected by high compression. This technique is based on the insertion of small diameter reinforced bars (stainless steel or FRP bars or plates) into the joint previously excavated and then repointed by mortars having generally better characteristics than the original ones (Modena et al., 2002; Valluzzi et al., 2005). This method can be adopted in case of towers subjected to sustained heavy loads.

2.4.2 BASE ISOLATION

The safety verification of the seismic action is often not satisfied in historical buildings and high seismic hazard zones. Seismic isolation is ensured using devices that give to the bearing system a large vertical stiffness and a low horizontal stiffness, which absorbs and reduces significantly the accelerations imposed by the earthquake. The isolated structure translates as a rigid body on the seismic isolation system, where all the deformations are concentrated. The periods increase and the structure leaves the area of the response spectrum with greater accelerations, while the displacements between floors are reduced. There are 3 families of devices: 1) Isolation: disconnection of the structure from the foundations, so that the seismic action transmitted by the ground does not reach the building. This device is equivalent to increasing the fundamental period of the structure. 2) Energy dissipation: limitation of the accelerations transmitted to the structure, by reducing the inertial forces. These devices absorb a part of the energy that acts on the structure. 3) Connection: in order to prevent or limit the displacements of the building with respect to the bearings, or in order to equally distribute the dynamic forces.

2.4.3 OTHER TECHNIQUES

It is clear that the seismic devices do not have to increase the stresses in the structure. The intervention can affect the all structure only where it is more vulnerable. A technique of seismic isolation can be made with an independent metal structure inserted inside the tower and linked to the walls (Barteletti et al., 1992; Balduzzi et al., 2006). For instance in the intervention in the bell-tower of Santa Lucia in Serra San Quirico in Ancona (Cosenza et al., 2007), an innovative material like FRP (Fiber Reinforced Polymers) was used to create a reticular system with vertical, horizontal and diagonal bands of FRP in the internal masonry of the tower (Figure 9a). These bands are 0.336 mm thick and 20 cm large.

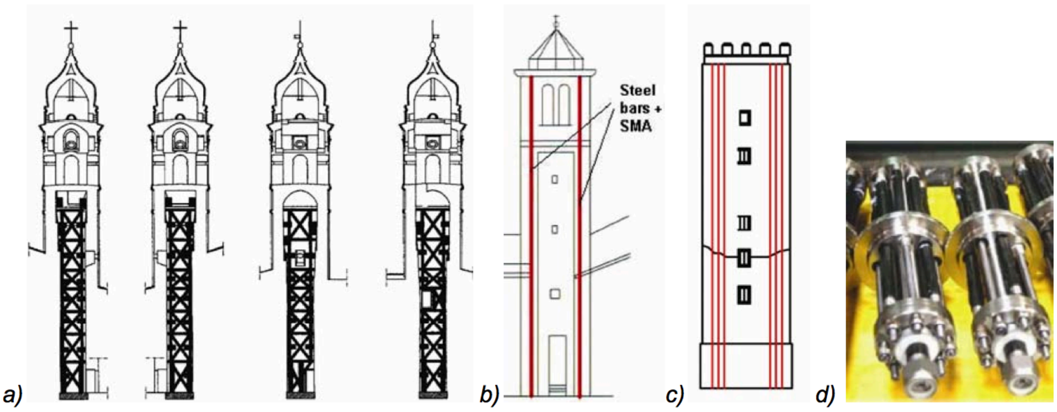


Figure 9 – Seismic strengthening: a) reticular System with FRP, bell-tower of S. Lucia (Cosenza et al., 2007); b) devices SMAD, bell-tower of S. Giorgio (Indirli, 2000); c) devices SMAD, Capocci Tower (Abruzzese et al., 2004); devices SMAD (Abruzzese et al., 2004).

Other innovative technique consists of SMAD (Shape Memory Alloy Device). During the research project ISTECH “Development of Innovative Techniques for the Improvements of Stability of Cultural Heritage, in particular Seismic Protection” the applicability of the SMAD devices has been verified. This project has been concluded in 1999 and it included the restoration of the bell-tower of San Giorgio in Trignano, close to Reggio Emilia, that was significantly damaged after the earthquake of 1996 (Indirli, 2000). In the repairing intervention steel pre-tensioned bars positioned vertically in the four corners of the bell-tower have been inserted (Figure 9b). Each tie rod is made with two segments divided by a SMAD device, which is at the third level of the structure. The same technique has been used in the seismic improvement of the Tower of Capocci in Rome (Abruzzese et al., 2004). To increase the assessment of the structure with the foundation and to increase the vertical tensile strength five tie rods at each internal corner have been inserted, with a SMAD device (Figure 9c).

3. LAMBERTI TOWER

The Lamberti Tower is a medieval tower high 84 m in Verona, northern Italy. This historical construction is located in the center of Verona, in "Piazza Erbe", and is the symbol of the city's municipal power. This tower, even if it has bells, is not a bell-tower, because it does not have any religious meaning: it is only the demonstration of the municipal power of the city.

Figure 10 – Lamberti Tower, picture from the square "Piazza Erbe"

In Italy, particularly in the medieval age, many towers appeared in the most important and powerful cities. The ancient builders tried to challenge the structural stability and the nature itself realizing towers, as a need of cities or families to show their power.

Fortunately the Lamberti Tower did not have particular damages in the course of the years. The tower is almost intact and in the last centuries only a few interventions have been carried out. The construction begun in 1172 and it is divided in two different phases. The first part of the tower, made with alternating layers of tufa and brick, has been done at the end of the 12th century and it corresponds more or less with the original tower. In May 1403 a lightning bolt struck the top of the tower, but only in 1448 restoration works were started and they continued until 1464. In this occasion the tower was enlarged: the more recent sections can be recognized today by the use of different materials. So the upper part has been built between 1448 and 1464. The large clock was added in 1779. The tower has been built with different materials: marble, bricks, tuff and stone. In the first part there is a combination of tuff and bricks, the second one only in bricks and finally only marble. In the first belfry there are “trifore” with columns and small stone balconies. Above there is the octagonal stone belfry with “bifore” and a stone perimeter frame. The roof is realized with eight inclined surfaces and there is a timber frame that supports lead plates. The tower is the highest of the city and reaches 79,10 m at the top of the roof and 83 m with the weathervane surmounting the sphere.

Figure 11 – Front and back of Lamberti Tower

3.1 GEOMETRY

The tower has depth of the foundations that is 5,20 m below the ground level. The plan dimensions are of 9.10 x 8.75 m (at the level of the courtyard of “Mercato Vecchio”) with thick walls of 2.30 m. The dimensions are gradually tapering going on the top of the tower. The plan dimensions become 8.70 x 8.50 m with a thickness of the wall of 1.80 m (at the level of 59 m from the ground). The first belfry has an octagonal base and is at 53.43 m, the second belfry is at 68.23 m and is supported by the previous one with a domed vault. The covering is at 77 m and has a pyramidal shape and octagonal base. Three openings adorn the top of the tower: monofore at the level of the clock, trifore at the level of the first belfry with stone columns and cantilevers which form four balconies, and bifore at the level of the second belfry with only one stone column in the center.

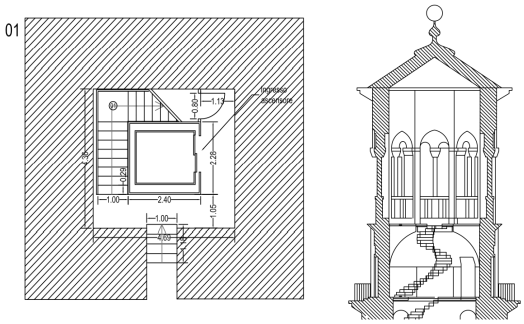


Figure 12 – Section of Lamberti Tower at the level of the courtyard “mercato vecchio” and on the right section of the top of the tower

3.2 CHARACTERISTICS OF THE SOIL

In December 2005 a soil survey by “TECNOBREVETTI s.r.l.” has been made. The result of the stratigraphy (made in the southeast corner of the tower considering the Geotecnica Veneta) has been: from 0.00 to -0.50 m flooring material; from -0.50 to -1.20 m fine gravel, from -1.20 to -1.90 m fragments of limestone with remains of bricks; from -1.90 to -3.50 m fine gravel with traces of clay and pebbles of porphyry; from -3.50 to -4.00 slightly silty sand with coarse gravel and pebbles of porphyry.

Figure 13 – Samples of the soil survey of Lamberti Tower

Results from other soil surveys carried out close to the Lamberti Tower have a very similar stratigraphy and show that the coarse gravel extends up to a depth of -15 m. Because the gravel has a very high load–bearing capacity it can be established that the soil and the foundations perform very well their static–dynamic task.

3.3 CHARACTERISTICS OF THE MATERIALS

The Lamberti Tower is a masonry tower divided in two main parts: until the level of 21 m it is made with courses of bricks and tuff and above the level of 21 m the masonry is composed of bricks. The small columns, which are possible to see in the belfry, are in compact stone and the covering is composed with timber. The foundations are made with stone masonry.

	Foundations with stone masonry	Masonry: course of bricks and tuff up to the level of 21 m	Masonry with only bricks above the level of 21 m	Small columns in compact stone
Compressive strength	f_k = 5 N/mm^2	f_k = 2.7 N/mm^2	f_k = 3.5 N/mm^2	f_k = 30 N/mm^2
Tensile strength	f_t = 0.3 N/mm^2	f_t = 0.2 N/mm^2	f_t = 0.3 N/mm^2	f_t = 0.4 N/mm^2
Modulus of elasticity	E = 8000 N/mm^2	E = 2700 N/mm^2	E = 3400 N/mm^2	E = 8000 N/mm^2
Shear modulus	G = 500 N/mm^2	G = 600 N/mm^2	G = 800 N/mm^2	G = 1200 N/mm^2
Coefficient of Poisson	ν = 0.025	ν = 0.00	ν = 0.00	ν = 0.025
Density	γ = 200 N/mm^3	γ = 180 N/mm^3	γ = 180 N/mm^3	γ = 200 N/mm^3

Table 2 – Characteristics of the material that compose the Lamberti Tower

3.4 DESCRIPTION OF THE BELLS

The bells of the tower are also a symbol of the city of Verona. The first bells were placed around 1272-1276 and over the years have been redone and replaced many times. The main bells are called “Rengo” and “Marangona”, being the first one bigger and being located above the other one. The sound of the bell “Rengo” had the principal aim to convene the city parliament called “Arengo”: it is from this word that derives the name “Rengo” of the bell. The goal of the bell “Marangona” was to indicate the beginning and the end of the work activities. The two bells have been rebuilt in 1311 for the important family of Verona “Gli Scaligeri” and after that there have been other restorations and recasting interventions.

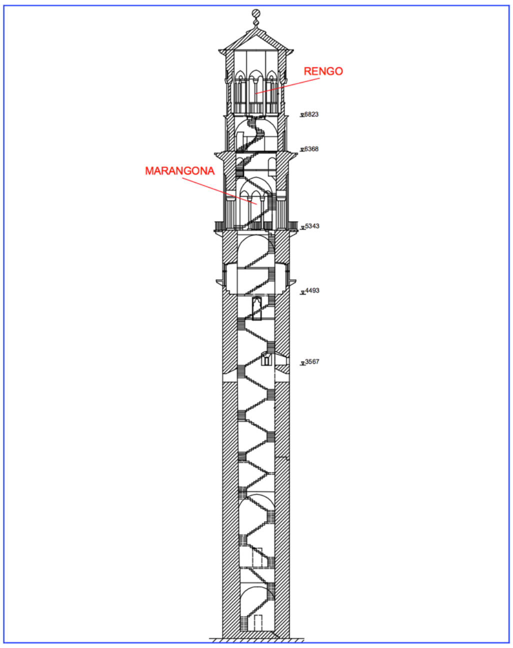


Figure 14 – Section of Lamberti Tower with the location of the two bells

The bell called “Rengo” is positioned on the top of the tower and has bigger dimensions when compared with all the others. This bell has a weight of 4215 kg, a diameter of 1831 mm and its direction of oscillation is North–South. It is the second biggest bell in the Italian region Veneto. The present bell has been placed in 1798, the same year that in the tower was inserted a clock. Today, the bell is used during major city events.

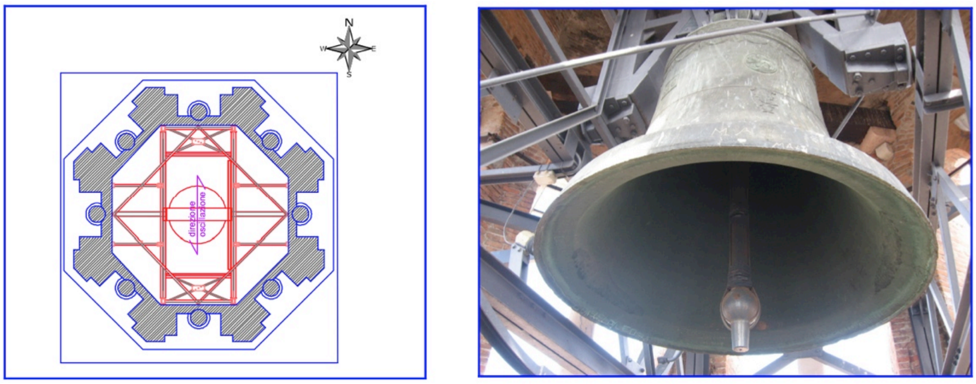


Figure 15 – Plant of the belfry and picture of the bell called “Rengo”

The bell called “Marangona” is positioned below the other bell “Rengo”. Two other smaller bells, called “Rabbiosa” and “SS. Fermo e Rustico”, flank it. The four bells together create a harmonious musical chord that was very famous in the music of the past. The frame of the belfry is in steel, with cast iron counterweights; unlike the frame “Rengo” this one does not have posts and the supporting beams are inserted directly into the walls where they are fixed with cement mortar. The bell “Marangona” has a weight of 1300 kg and a diameter of 1298 mm, the bell “Rabbiosa” has a weight of 750 kg and a diameter of 1080 mm and finally the bell “SS. Fermo and Rustico” has a weight of 330 kg and a diameter of 610 mm. All these bells have a direction of oscillation that is East – West.

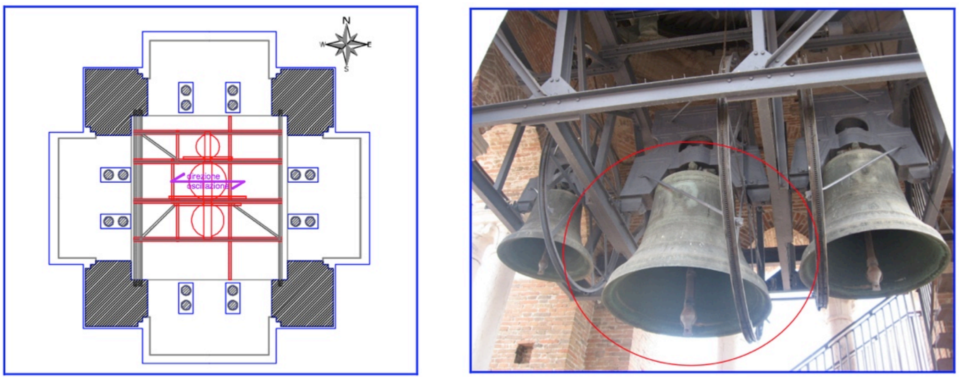


Figure 16 – Plant of the belfry and picture of the bell “Marangona” flanked by the other two smaller

3.5 PREVIOUS DYNAMIC ANALYSIS RESULTS AND INSPECTIONS

The office “4-EMME SERVICE S.p.a.” carried out preliminary analyses on the Lamberti Tower. The aim of their project has been the verification of the safety conditions of the tower under excitation of the bells. A Dynamic Analysis was made and with the appropriate tools the natural frequency of the tower has been obtained by exciting the structure with a few tolls of the two biggest bells. The induced vibration has been obtained by the movement of the bell called “Rengo” in direction north – south and the bell “Marangona” in direction east – west.

Figure 17 – Accelerometers positioned in the N-W and S-E corners of the belfry “Rengo”

For the determination of the natural frequency of the structure it has been used three pair accelerometers. The sensors were placed respectively in the north-west and south-east corner (on the opposite sides of the octagonal belfry of the "Rengo") on the reinforced concrete frame which is 68.50 m from ground level. The second backhoe of accelerometers was placed on the balcony of a small opening, at east more or less in the middle of the tower (33.50 m from ground level). The sensors have been placed in the position shown by Figure 18.

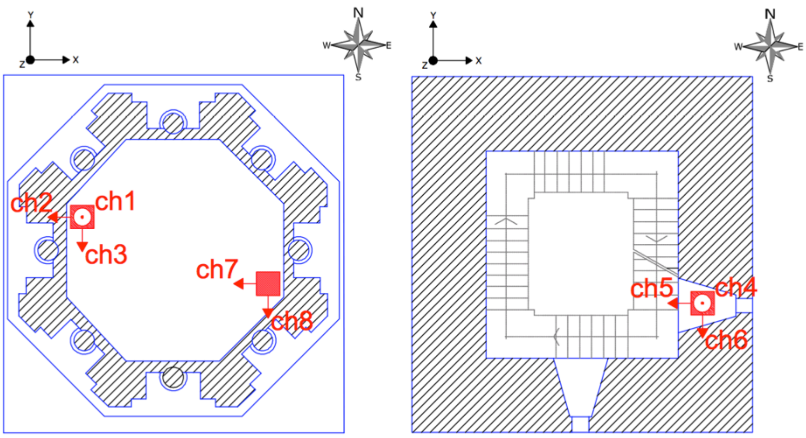


Figure 18 – Accelerometers in the belfry “Rengo” (ch1, ch1, ch3, ch7 and ch8) and in the middle of the tower (ch4 to ch6)

Therefore, as already mentioned previously, the purpose of the analysis of the belfry has been to evaluate the total action of the movement of the four bells. The dynamic stresses produced by the vibration of the bells are usually, both considering displacements and stresses, the principal reasons of damages in towers. The effects can be localized, for instance cracks in the belfry due to the vibrations of the bronze, or diffuse, for example the resonance phenomenon.

Usually the bells are generally blended with a binary alloy of copper (78%) and tin (22%), commonly known as bronze (density ρ = 8580 kg / m 3). Copper gives the "hardness" and tin the "sounds". The sound and vibration due to the bell forward to the surrounding medium (air) once the bell is hit by the clapper. This vibration, however, is also transmitted to the frame, which supports the bell and if the frame consists of metallic material such as steel, it transfer the vibration to the masonry in the respective anchor points. The effect becomes a progressive crumbling of the cement mortar, which entails a reduction of the mechanical properties of the masonry, with the consequent occurrence of cracks even if located in one cell bell. It is important, even when the frame is fixed to the structure, to provide some heat sinks such as rubber pads, usually made of neoprene. This solution has not been made for both the frame of "Rengo" and for the frame of the other bells.

In Figure 19 there are two windows with 20 seconds of the oscillogram of the channel X1 e X3 in direction E-W and the results of the channels Y1 and Y3 in the direction E-W in the “Rengo” belfry. In the Figure 19 is presented the spectrogram of the signals is direction E-W and N-S obtained by the movements of the all the channels.

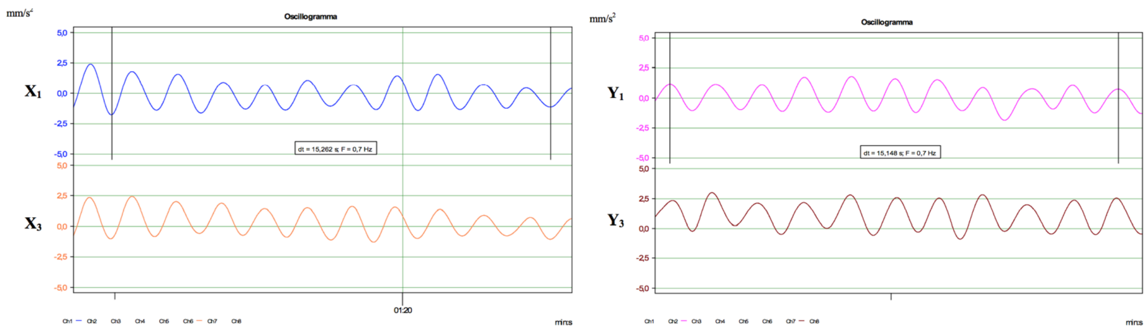


Figure 19 – Channels oscillogram in the direction E-W (on the left) and in the direction N-S (on the right)

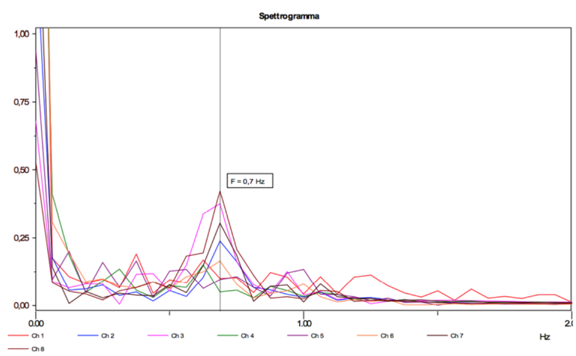


Figure 20 – Spectrogram of all the channels

From these results is possible to assume that, without considering the direction of the vibration, the frequency is the same in both directions N-S (Y) and E-W (X). After a previous dynamic analysis by the office “4-EMME SERVICE S.p.a.”, another office “Tecnobrevetti s.r.l.” has performed a mathematical model with FEM using 3D elements to obtain a better response of the state of stress in the tower. For the masonry and for the foundations “BRICK” elements with 8 nodes have been used, for the small stone columns of the belfry “BEAM” elements with 2 nodes have been used and for the covering in lead plates supported by timber frame “PLATE-SHELL” elements with 4 nodes have been used. The results of the numerical processing of the model carried out by “Tecnobrevetti s.r.l.” are shown in Table 3.

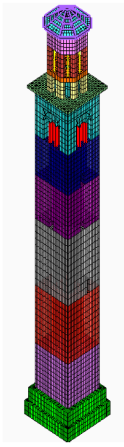


Figure 21 – Model of Lambert Tower made by “Tecnobrevetti s.r.l.”

MODE	Modal Analysis: BRICK MODEL		
	Frequency (Hz)	Period T (s)	Number od oscillations (min^{-1})
Mode 1	0.6935 Hz	1.44	41.61
Mode 2	0.7195 Hz	1.389	43.17
Mode 3	2.3143 Hz	0.43	138.86
Mode 4	2.6357 Hz	0.38	158.142
Mode 5	2.69035 Hz	0.37	161.42

Table 3 – Principal natural frequencies of the 3D Model (Tecnobrevetti s.r.l., 2008)

In Table 3, next to each mode, there is the principal direction of oscillation. The first two modes are the deformed cantilever (single curvature) according to the main axes, where the second mode shows a lower period because the section of the tower is not a perfect square, the third one is torsional and the others are the deformed with higher curvatures.

In the Dynamic Analysis Report of "Tecnobrevetti s.r.l." the analysis of the actions of the bells has been simplified, considering that the movement is independent of the displacements of the tower (separate dynamic analysis). Therefore an input that describes, as a function of time, the stress that each bell produces on their supports has been defined. Subsequently, the response of the belfry of the tower has been obtained. With the purpose of determining the amplitude and the time distribution of the stresses, a bell can be simplified as a compound pendulum with length l (equal to the distance of the barycenter of the total mass m from the pivoting axis) and subjected only to its weight. The position of the barycenter G is found from the value of the angle $\boldsymbol{\theta}$ from the curvilinear abscissa s and the relation $\boldsymbol{s} = \boldsymbol{l} \cdot \boldsymbol{\theta}$ is valid. The analysis of the data obtained allows observing firstly that each belfry is mostly affected by the vibrations of its own bell. The effects of the bell "Marangona" for the belfry of "Rengo" and vice-versa are negligible compared to the effects due to their effect on the structures of the respective belfry. Therefore, the analysis has been made considering the effects due of the movement of all the bells independently.

The assessment of vibration hazard is regulated by UNI 9916-2004 that proposes the acquisition of experimental data and indicates the possible damage in the structures, considering the velocity limits and peak acceleration as a function of frequency. The buildings are classified according to three categories:

- Category I: Buildings with adequate foundations or supported by piles, constituted by structural elements in reinforced concrete, steel or timber.
- Category II: Masonry construction or buildings where masonry is the prevalent material.
- Category III: Buildings of category II with considerable age or of cultural importance, and buildings that are not in perfect condition.

The Lamberti Tower is in masonry, very old and with an historical importance, therefore it has been classified in category III, even if it is in good condition.

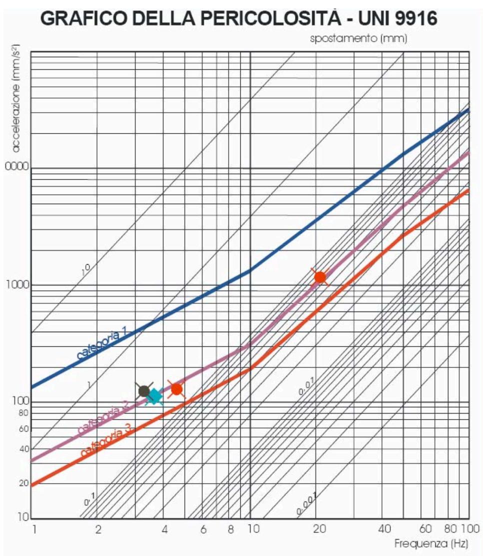


Figure 22 – Summary of the results. There is: top of x axis - displacement (mm) / below of x axis - frequency (Hz) / y axis - acceleration (mm/s)

Therefore, two approaches have been adopted to assess the safety of the tower with respect to the bells action. The first approach has been a theoretical analysis based on a mathematical model using finite elements to evaluate the stress state on the significant points of the structure. The second approach has been an experimental evaluation of the structural response based on measurement of acceleration peaks in the points.

From the first analysis it has been possible to observe that the action of the bells provide tensile stresses rather small compared to the values obtained in the analysis of the tower due to self-weight. The action of the three lower bells is more dangerous than the action of the upper bell, and this probably due to the fact that the upper bell is better balanced than the others. The stresses on the steel frame most affected by the vibrations of the bells are also rather low. The evaluation of the masonry walls has been different: even if the stresses are modest, they may create a deterioration of mortars and therefore create damage in the masonry or in the stone materials of the small columns that adorn the belfries. The stresses and displacements on the rest of the tower have been considered negligible because they are absorbed and damped, not evening reaching the base of the tower.

From the second analysis it has been possible to observe that only in three points (two for the top belfry and one for the lower belfry) the safety levels exceeding those indicated in the Italian code for Category III, even if acceptable for Category II. The report suggests monitoring during the years the Lamberti Tower, especially with regard to the belfries, which are the most vulnerable parts of the structure. It is also recommended to control the evolution of possible damage and, if present, relate it with the dynamic action of the bells.

4. DYNAMIC IDENTIFICATION

Dynamic identification is a technique used to perform an experimental modal analysis of a structure in order to get its dynamic response. The response of the structure is usually analyzed in the frequency domain while the measures registered by the data acquisition system are often accelerations, in few cases velocities and almost never displacements. The entire process to determine the experimental modal parameters can be considered composed of two different parts. The first step is given by the experimental test performed thanks to the installation of a data acquisition system, while the second step is the determination of the modal parameters by means of different available analytical methods.

The process of experimental investigation is a necessary step to obtain a reliable diagnosis of any historical masonry building. For this reason it is advisable to collect as many information – related to the structural behavior of the building – as possible. In fact, it can be considered hardly possible to develop fully realistic, reliable behavioral models of real" historical masonry structures, in absence of experimentally verified data.

In comparison with different non-destructive techniques, dynamic identification is conceptually very interesting: while other methodologies generally provide qualitative or quantitative information on local properties of the constituting materials or structural elements, dynamic testing is possibly the only way to experimentally measure parameters related to the global structural behavior. In fact, dynamic identification tests are intended for the characterization of the modal response of the building, being the obtained results referable to several structural/physical parameters, such as mass distribution, geometry stiffness and boundary conditions. On the contrary, the contribution of the technique to a clear understanding of what is going on in a historical structure in terms of presence of damage and, in particular, its propagation, can be considered limited.

The possibility to use a natural source of excitation to define the modal response of a historical structure is very appealing, considering that the procedures used to provoke a detectable dynamic response in new structures are often unviable for the historical ones, and tests inducing structural resonance are to be carried out with much precaution. Using ambient vibrations allow not to harm under any circumstance the historical building, a key point in the preservation effort that involves also the investigation phase, and test costs are reduced to the data acquisition activities, which are relatively fast and standard. It is noted that the final scope of the present research is the evaluation of the seismic response of the building, and this testing methodology only provides information on the dynamic behavior of the system in its initial state (usually considered elastic). Still, the contribution of the ambient modal analysis in terms of model updating, especially when combined with calibrations carried out with other experimentally measured “static” parameters (such as local Young modulus and local stress, via flat-jack measurements) can be very interesting for the “qualitative” interpretation of structural faults.

Modal Analysis, or more accurately Experimental Modal Analysis, is the field of measuring and analyzing the dynamic response of structures when excited by an input. It combines vibration test data and analytical methods to determine modal parameters of a structural system. Dynamic properties resulting from experimental modal analysis are the frequencies, damping and mode shapes, related to the physical and mechanical characteristics of the analyzed structures (mass, stiffness and energy dissipation). In case of historical constructions, Modal Analysis can be adopted for the evaluation of dynamic characteristics of buildings (natural frequencies and modes), the validation of behavioral models (typically FEM), troubleshooting of structures experiencing problems in response, or control system for structures (monitoring).

4.1 MODAL ANALYSIS

The dynamic identification analysis has been performed on May 29th, 2013 by a team from the Department of Civil Engineering of the University of Padova, Italy. The acquisition was carried out considering ambient vibration (OMA, Operational Modal Analysis), consisting in wind excitation, human activities, the bells ringing on the top (every 30 minutes), and the presence of an elevator up to the level of 54.43 m (the first belfry). The aim of the acquisitions was to define the modal parameters (natural frequencies and mode shapes) of the building in order to update a reference Finite Element (FE) model.

4.1.1 TEST PLANNING

To measure the dynamic response of the Lamberti Tower to ambient vibrations, four levels of the structure have been selected. The measuring equipment was composed by 5 uniaxial piezoelectric accelerometers, with a bandwidth ranging from 0.15 to 1000 Hz, a dynamic range ±0.5 g and a voltage sensitivity of 10000 mV/g,

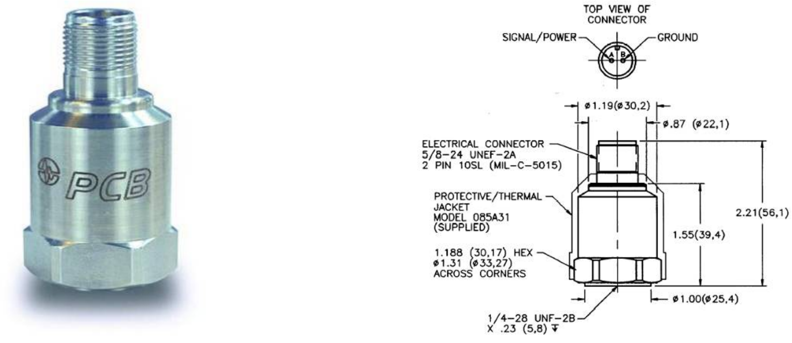


Figure 23 – Accelerometer model 393 B12, picture and technical details

The accelerometers were connected to a data acquisition system with 16 bit A/D converter, provided with anti-aliasing filters in the amplification cards. As the digital band was configured to digitalize between ±0.05 g, the system resolution was equal to 8 μg (equal to the transducer resolution). Two cables connected the accelerometers to the computer workstation with a data acquisition board for A/D and D/A conversion of the transducer signals and storage of digital data. For each channel the signals converted to digital form were stored on the hard disk of the data acquisition computer in ASCII form. Figure 24 shows the acquisition system that was placed at the level of 44.93m.

Figure 24 – Acquisition System multi-channel, NI mod. PXI-1025 MegaPAC

The sensors were placed in correspondence of the four corners of the rectangular section of the tower; a drill was used to make two small holes, in the mortar between the bricks, to connect each accelerometer to the masonry. Due to the large dimensions of the Tower and the availability of 5 biaxial accelerometers, the use of different setups was necessary to get a dynamic characterization of the building as better as possible.

For the dynamic identification 4 setups were planned for 16 total points of data acquisition (Figure 25).

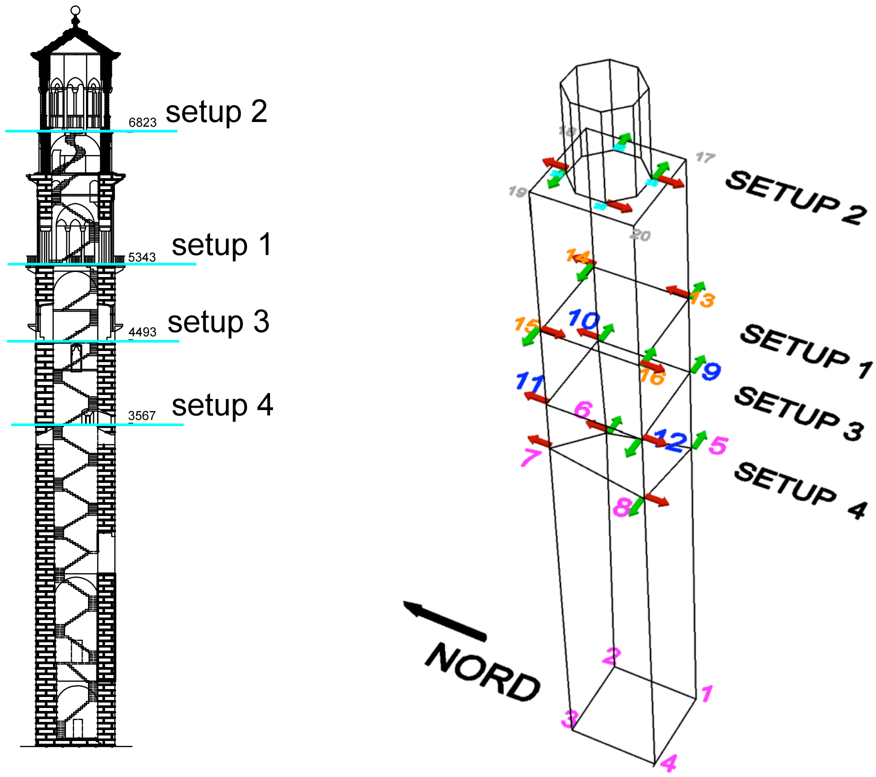


Figure 25 – Setup plan and measuring points with different directions

The first setup was placed in the first belfry, at the level of 53.43 m, and 5 registrations were recorded. The second setup, with 4 registrations, was chosen in the second belfry, the upper part of the tower at the level of 68.23 m (the second octagonal belfry). The third setup was located at the level of 44.93m, where the acquisition system was placed, and 5 acquisitions were performed. The last setup was chosen at the level of 35.67 m, where there are some openings, and each sensor was positioned in a different height of the tower: respectively at 33.67 m, 35.67m, 37.67m and 39.76 m. At this last step 4 registrations were acquired.

Keeping one accelerometer fixed (reference sensor) and changing the position of the others it was possible to obtain results from more points and have a full identification of the tower. The reference sensor has the function of point of control and it allows the correlation and the combination of the different results. The reference channels, during the dynamic tests of Lamberti Tower, were located in correspondence of the corner S-W (Figure 26). Figure 27 shows an example of an accelerometer before the test and Figure 28 during testing.

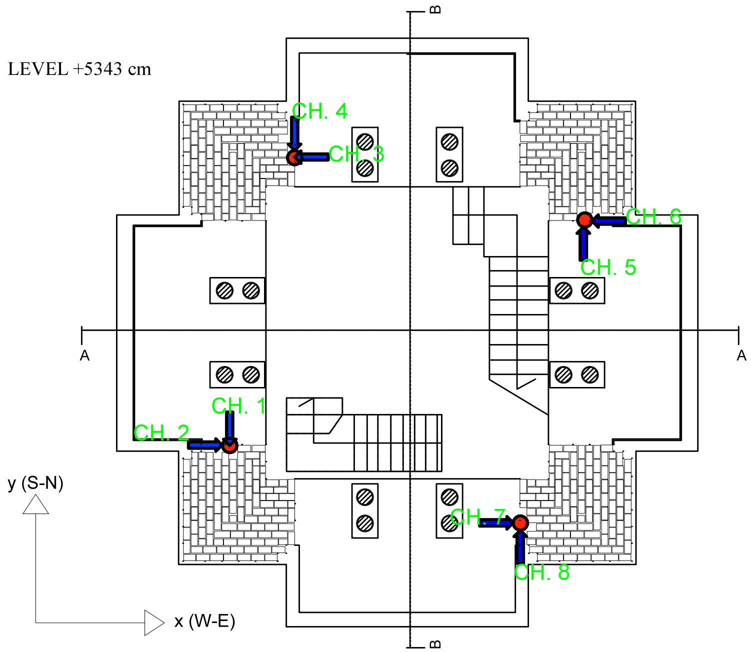


Figure 26 – Direction of the channels of SETUP 1, at the level of the first belfry 53.43m

For each setup the registrations were approximately 10 minutes long and the sampling rate was 100 sps (samples per second). Accelerations were acquired in two perpendicular directions: north/south and east/west. The ambient vibrations were measured in all of the 16 points with 4 sequential setups.

Output-only modal identifications techniques were used to estimate the modal parameters: resonant frequencies, mode shapes and damping coefficients. These techniques are based on the dynamic response measurements of a virtual system under natural (ambient or operational) conditions, and they are based on the assumption that the excitation is reasonably random in time and in the physical space of the structure.

Figure 27 – Accelerometers 1,2

Figure 28 – Sensors 1,2 connected with cables

4.1.2 EXPERIMENTAL RESULTS

The signal processing was carried out using the software ARTeMIS Extractor Pro. The Stochastic Subspace Identification (SSI) method, Unweight Principal Component (UPC), was adopted as it is robust and allows modal parameters estimation with high frequency resolution. SSI method is described in the previous section. Although frequency domain methods are fast in processes and easy to interpret, SSI methods give more accurate results especially when the frequencies are close to each other and difficult to distinguish from other outputs (noise).

As it can be seen in Figure 29, high model order of the input matrix is selected where the estimation of frequencies gets constant and constitutes straight vertical lines called as poles. In each setup, the model order was chosen manually. Frequency estimation results are given in Table 4. The first mode appears at 0,65 Hz where also occurs the higher amplitude occurs.

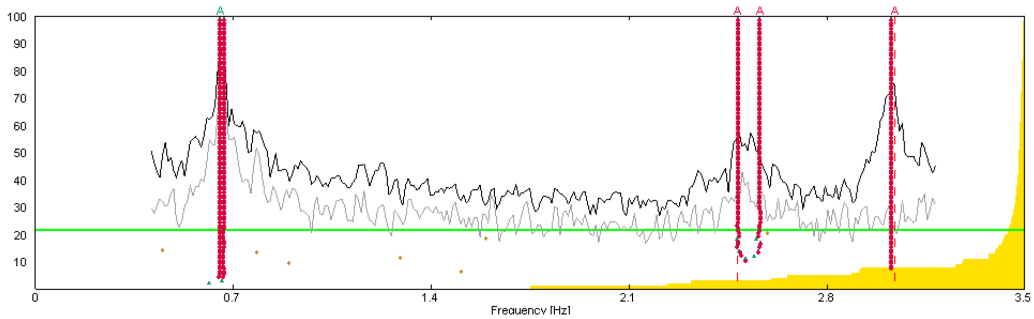


Figure 29 – First 5 natural frequencies of Lamberti Tower

MODE	Frequency [Hz]
1	0.65
2	0.67
3	2.48
4	2.55
5	3.04

Table 4 – Estimated modes with the SSI method

During the signal processing all the registrations at each setup were analyzed. The signal was not clear and full of interferences (due to the bell ringing, the activity of the elevator and the presence of many tourists). The best registration for the first setup was the fourth that was cropped to clarify the signal. For the second setup, the second registration was chosen, even if all the 4 registrations were disturbed by the bell ringing. At the third setup the second registration was chosen and cropped. The signals of all the

4 registrations of the last setup were not good. The second signal for this set-up was chosen and cropped.

Some frequencies were automatically detected. Due to the proximity of the first two peaks, as shown in Figure 29, it was difficult to identify automatically the first two frequencies. A manual selection was therefore adopted (Figure 30) and it was possible to detect the main natural frequencies that are concentrated in the range between 0.6 Hz and 3 Hz.

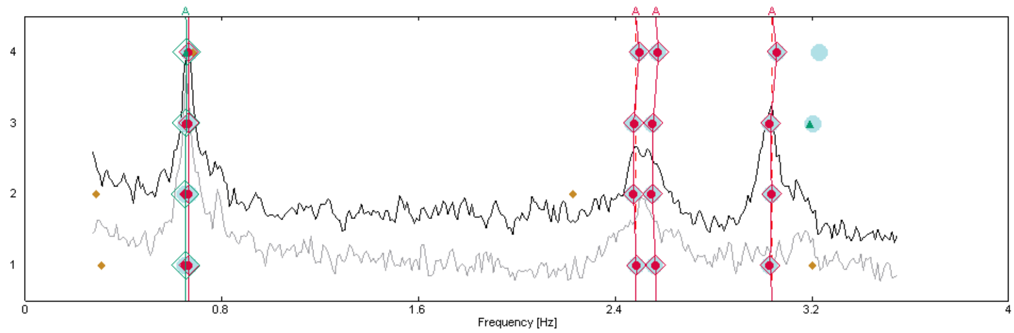


Figure 30 – First 5 natural frequencies of Lamberti Tower

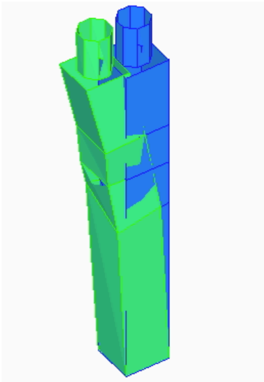
ARTeMIS allows a visual simplified representation of the modal shapes. In order to reach this purpose the construction of a schematic model of the structure and the definition of the measurement points with their directions are necessary. It is important to underline that in this schematic representation only the elements (points, line, surfaces) connected to the measured points will move.

The modes obtained are:

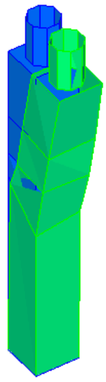
- Mode 1 (f = 0.65 Hz) : is the first fundamental longitudinal movement (N – S direction);
- Mode 2 (f = 0.67 Hz) : is the second fundamental longitudinal movement (E –W direction);
- Mode 3 (f = 2.48 Hz) : the movement is a bending and it assumes two curvatures. It seems not exactly parallel to one of the axis, but rotated 45°;
- Mode 4 (f = 2.55 Hz) : the movement is similar to the Mode 3 but in the opposite direction. This mode is also rotated 45°;
- Mode 5 (f = 3.04 Hz) : the movement is a little distorted but it seems torsion.

Figure 31 shows the first five mode shapes detected. The blue model represents the original geometry of the Tower, while the light green represents the deformed shape. The experimental mode shapes show that the tower response is almost symmetric. The cross section is a rectangular shape with one side slightly larger than the other. As a consequence, the first mode shape is bending of the tower around the smaller side of the section.

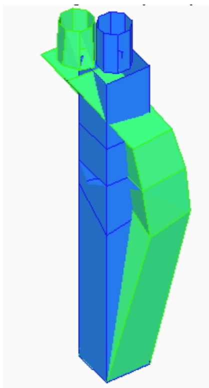
Mode Shape 1
Frequency= 0.65 Hz
Bending direction (N-S)



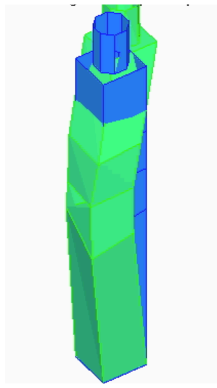
Mode Shape 2
Frequency= 0.67 Hz
Bending direction (E –W)



Mode Shape 3
Frequency= 2.48 Hz
Bending, direction (N –S)



Mode Shape 4
Frequency= 2.55 Hz
Bending direction (E –W)



Mode Shape 5
Frequency= 3.038 Hz Torsion

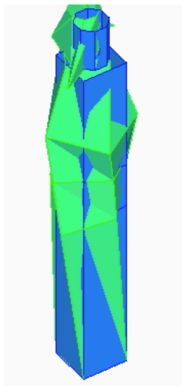


Figure 31 – First mode shapes detected by the dynamic identification test

5. DEFINITION OF THE NUMERICAL MODEL

In the structural analysis of cultural heritage historical buildings, the first step is usually the attainment of a sound knowledge of the structure and of the composing materials, in order to proceed with the assessment of the building with reliable data. The geometry used in the 3D model of the Lamberti Tower is based on drawings and documents of previous studies and the material characteristics are taken from a report published in the 2008 from the Italian Office “Tecnobrevetti s.r.l.” (Table 2).

Currently, advanced numerical analysis methods are being increasingly used in the design and assessment of structures. As the computer technology develops the efficiency of models are increasing. However, when the numerical predictions are compared with experimental results, the coherence may fall out of confidence limits (Friswell et al, 2009). For this reason a comparison between the results of the dynamic test and the numerical structural analysis is described in this chapter. Consequently, a model updating is carried out, in order to obtain a structural FE model closer to the real behavior of the tower. It is stressed that two different models were developed. The first model was made with 3-D composite beams (Beam model), while the second model was performed with 3-D solid elements (3-D model), being both models analyzed with the commercial finite element code DIANA.

5.1 THREE DIMENSIONAL BEAM MODEL

A simplified structural model considering the tower as a cantilever beam was performed. The model comprehends 10 class III beam-type elements, with quadratic interpolation and using the Mindlin-Reissner theory. The CL18B element (Fig. 32) is a three-node, three-dimensional class-III beam element. Basic variables are the translations in u_x, u_y and u_z and the rotations $\phi_x \ \ \phi_y \ \ \phi_z$ in the nodes.

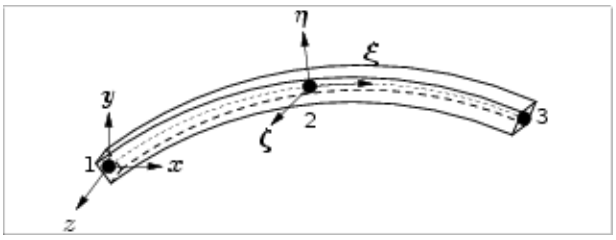


Figure 32 – CL18B Beam element

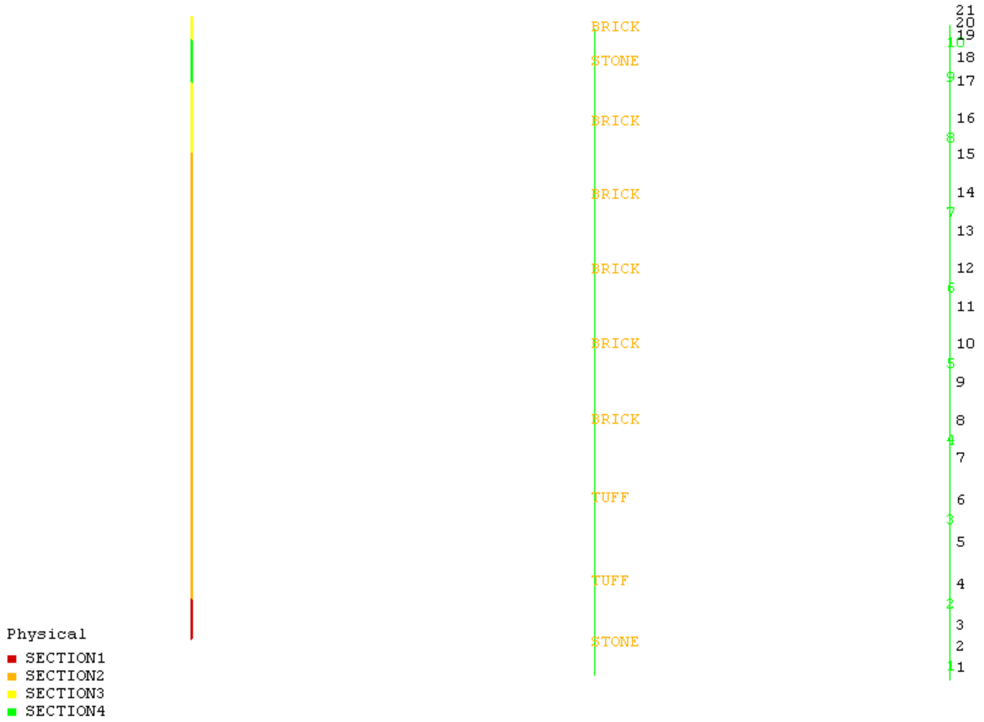
The interpolation polynomials for the displacements can be expressed by the equation 17:

$$u_i(\xi) = a_{i0} + a_{i1}\xi + a_{i2}\xi^2 \quad (17)$$

$$\phi_i(\xi) = b_{i0} + b_{i1}\xi + b_{i2}\xi^2$$

Due to these polynomials the strains vary linearly along the center line of the beam. The software DIANA applies a 2-point Gauss integration scheme along the bar axis. Each element has different physical properties that correspond to the geometry and the characteristics at each level of the tower. The first section corresponds to the foundation of the tower (Figure 33). A predefined box shape with 9.12 m height, 9.15m width and a thickness of 2.3m in all the four lateral sides was chosen. The second section represents the lower part of the tower made with courses of bricks and tuff; the geometry, inserted again in the predefined box shape, has 8.76m height, 9.08m width and a thickness of 1.90m. Above the level of 21 m, the masonry of the tower is composed of bricks and the cross section was defined in the model with the same geometrical characteristics of the previous part (see Figure 33a "section2"). From the level of 59.00 m, a smaller cross section was defined. The values introduced to define this box shape are 8.50 m height, 8.70 m width and 1.80 m thickness of the lateral walls. The last section represents the belfry on the top of the tower. In the model, a predefined pipe shape was chosen, with a diameter of 7.8 m and a thickness of 0.8 m.

In the structural modal analysis the characteristics of the materials, described in the previous Italian report made by "*Tecnobrevetti s.r.l., 2008*" (see Table 2 in chapter 3), were chosen. The materials defined in the model are represented in the figure 33b. The timber roof was not considered in the beam model because it is irrelevant for this structural model. In the figure 33c, it is possible to see that the final model is composed by 10 beam elements and 21 nodes. As boundary condition, the base point of the tower was fully restrained.



(a) (b) (c)

Figure 33 – Beam Model: a) different sections used at each level of the model; b) materials defined eith different physical properties; c) numbers of the nodes and of the elements.

5.2 EIGENVALUE ANALYSIS OF THE BEAM MODEL

The mode shapes obtained from the eigenvalue analysis of the 3-D beam FE model are the same of the experimental test in terms of configuration, but not in terms of frequencies (Figure 34). It was chosen to present only the first five numerical modes in order to make a comparison with the experimental results.
A comparison between the experimental frequencies and the frequencies obtained with the 3-D beam FE model is presented in Table 5. It is possible to observe that there is a clear difference between the experimental and the numerical results. The frequencies obtained by the Italian Office “Tecnobrevetti” are also shown. The comparison shows that the results of the solid model *Tecnobrevetti,* made previously in 2008, are much closer to the experimental values. In the present study, the same geometry and the material characteristics that *Tecnobrevetti* chose in their solid model but it was not possible to obtain results. For this reason, an additional simplified structural model, considering the tower as cantilever beam, was performed with a different commercial finite element software, STRAUS7, in order to compare the fundamental frequencies. The results are presented in the Table 6.

EXPERIMENTAL FREQUENCIES			NUMERICAL FREQUENCIES BEAM MODEL		EXPERIMENTAL FREQUENCIES Tecnobrevetti, 2008	
Mode 1	0.65 Hz	Bending1	0.36 Hz	Bending1	0.69 Hz	Bending1
Mode 2	0.67 Hz	Bending2	0.373 Hz	Bending2	0.72 Hz	Bending2
Mode 3	2.48 Hz	2° ord. B1+B2	1.99 Hz	2° ord. B1+B2	2.31 Hz	Torsion
Mode 4	2.55 Hz	2° ord. B1+B2	2.02 Hz	2° ord. B1+B2	2.63 Hz	2° ord. B1+B2
Mode 5	3.03 Hz	Torsion	2.08 Hz	Torsion	2.65 Hz	2° ord. B1+B2

Table 5 – Comparison between the experimental frequencies, 3D beam model frequencies and the frequencies from the Italian

Mode Shape 1
Frequency= 0.3567 Hz
Bending, direction (N-S, x-x)

Mode Shape 2
Frequency= 0.3673 Hz
Bending, direction (E-W, y-y)

Mode Shape 3
Frequency= 1.9899 Hz
Bending, direction (N-S, x-x)

Mode Shape 4
Frequency= 2.022 Hz
Bending, direction (E-W, y-y)

Mode Shape 5
Frequency= 2.0802 Hz
Torsion

Figure 34 – Modal shapes and frequencies of the BEAM FE mode from DIANA software

NUMERICAL FREQUENCIES FROM BEAM MODEL *DIANA*			NUMERICAL FREQUENCIES FROM BEAM MODEL *STRAUS7*	
Mode 1	0.36 Hz	Bending1	0.34 Hz	Bending1
Mode 2	0.37 Hz	Bending2	0.35 Hz	Bending2
Mode 3	1.99 Hz	2° ord. B1+B2	2.34 Hz	2° ord. B1+B2
Mode 4	2.02 Hz	2° ord. B1+B2	2.65 Hz	2° ord. B1+B2
Mode 5	2.08 Hz	Torsion	2.73 Hz	Torsion

Table 6 – Comparison between the 3D beam model frequencies obtained with the software DIANA and STRAUS7

The values obtained with the two different finite element software DIANA and SRAUS7 are similar, with increasing differences for higher modes. This is possibly due to the fact that different beam elements are considered in the two softwares. Observing the direction of the mode shapes, it is also interesting to see that the modes of the two Beam FE models have the same deformed shape of the experimental results. In fact the first mode is bending with single curvature in N-S direction while the second mode is also bending but in E-W direction; the third mode is a second order bending in N-S direction, the fourth is still a second order bending in E-W direction. The fifth mode is a torsion mode around the Z axis. This is not true for the structural analysis of the FE solid model from *Tecnobrevetti2008* where the torsion is in the third mode (see Table 3).

In conclusion, the beam models allowed to state that two different softwares provided a similar result between themselves and mode shapes similar to the configurations obtained experimentally. The frequencies obtained in the models are much different from the frequencies obtained experimentally, meaning that the boundary conditions or material properties need to be fine-tuned. Another model for the same structure in the literature provides frequencies much different from the ones obtained with the 3D beam model. For this reason, a 3D finite element model of the tower was also prepared and is discussed in the next section.

5.3 THREE DIMENSIONAL SOLID MODEL

A three dimensional model with solid elements was performed. The model is made primarily by BRICK elements that compose the foundations, the lateral walls and the two belfries of the tower. The columns at the first and the second belfry were represented by 3-D truss elements and 3-D curved shell elements were chosen for the timber roof at the top of the tower.

The HX24L brick element (Figure 35) is an eight-node isoparametric solid brick element. It is based on linear interpolation and Gauss integration. The polynomials for the translations u_{xyz} can be expressed as:

$$\boldsymbol{u_i(\xi,\eta,\zeta) = a_0 + a_1\xi + a_2\eta + a_3\zeta + a_4\xi\eta + a_5\eta\zeta + a_6\zeta\xi + a_7\xi\eta\zeta} \qquad (19)$$

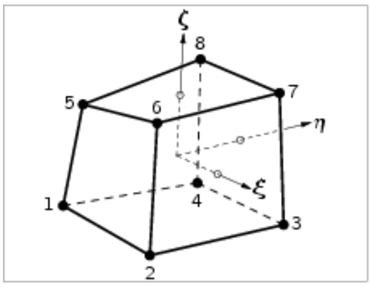


Figure 35 – HX24L solid element

Typically, a rectangular brick element approximates the following strain and stress distribution over the element volume. The strain ε_{xx} and stress σ_{xx} are constant in x direction and vary linearly in y and z direction. The strain ε_{yy} and stress σ_{yy} are constant in y direction and vary linearly in x and z direction. The strain and stress are constant in z direction and vary linearly in x and y direction. If bending behavior is dominant the use of incompatible bubble displacement modes is advised. By default DIANA applies a 2 x 2 x 2 $[n_\xi = 2,\ n_\eta = 2,\ n_\varsigma = 2]$ integration scheme.

The section of the belfry (see Figure 15 in Chapter 3) is an octagonal shape. For this reason the corners of the octagonal belfry were modeled as wedge elements. The TP18L element (Figure 36) is a six-node isoparametric solid wedge element. It is based on linear area interpolation in the triangular domain and a linear isoparametric interpolation in the ζ direction. The polynomials for the translations u_{xyz} can be expressed as:

$$\boldsymbol{u_i(\xi,\eta,\zeta) = a_0 + a_1\xi + a_2\eta + a_3\zeta + a_4\xi\eta + a_5\eta\zeta} \qquad (20)$$

These polynomials yield a constant strain and stress distribution over the element volume. By default DIANA applies a 1-point integration $(n_{1c} = 1)$ scheme in the triangular domain and 2-point $(n_\zeta = 2)$ in the ζ direction.

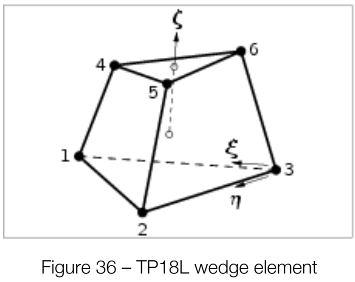


Figure 36 – TP18L wedge element

The roof was modeled using curved shell elements. These types of elements in DIANA are based on isoparametric degenerated-solid approach by introducing two shell hypotheses: (a) straight-normals, which assumes that normals remain straight, but not necessarily normal to the reference surface. Transverse shear deformation is included according to the Mindlin-Reissner theory; (b) Zero-normal-stress, which assumes that the normal stress component in the normal direction of a lamina basis is forced to zero. The element tangent plane is spanned by a lamina basis which corresponds to a local Cartesian coordinate system (x_1, y_1) defined at each point of the shell with x_1 and y_1 tangent to the ξ,η plane and z_1 perpendicular to it.

The in-plane lamina strains ε_{xx}, ε_{yy} and γ_{xy} vary linearly in the thickness direction. The transverse shear strains γ_{xz} and γ_{yz} are forced to be constant in the thickness direction. Since the actual transverse shearing stresses and strains vary parabolically over the thickness, the shearing strains are an equivalent constant strain on a corresponding area. A shear correction factor is applied using the condition that a constant transverse shear stress yields approximately the same shear strain energy as the actual shearing stress.

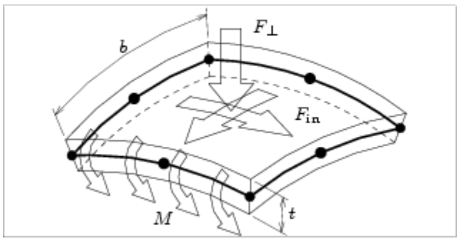


Figure 37 – Curved shell elements, characteristics

The triangular T15SH element was chosen for the model (Figure 37), which is a three-node triangular isoparametric curved shell element. It is based on linear interpolation and area integration. The integration in ζ direction (thickness) may be Gauss or Simpson. The polynomials for the translations u and the rotations ϕ can be expressed as:

$$\boldsymbol{u_i(\xi,\eta) = a_0 + a_1\xi + a_2\eta}$$
$$\boldsymbol{\phi_i(\xi,\eta) = b_0 + b_1\xi + b_2\eta} \tag{21}$$

Typically, these polynomials yield approximately the following strain and stress distribution along the element area in a ζ lamina. The strain ε_{xx}, the curvature k_{xx}, the moment m_{xx}, the membrane force n_{xx} and the shear force q_{xz} are constant in x direction and vary linearly in y direction. The strain ε_{yy}, the curvature k_{yy}, the moment m_{yy}, the membrane force n_{yy} and the shear force q_{yz} are constant in y direction and vary linearly in x direction. The default integration scheme over the element area is a 3-point scheme ($n_{1c} = 3$). The default in ζ direction (thickness) is 3-point Simpson, ($n_\zeta = 3$).

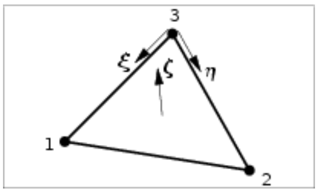


Figure 38 – T15SH shell element

The columns on the belfry of the tower were modeled with truss elements L2TRU, straight, 2 nodes were used (Figure 39), which is a two-node directly integrated (1-point) truss element. This polynomial yields a strain ε_{xx}, which is constant along the bar axis.

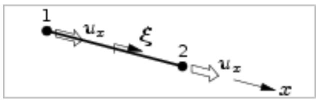


Figure 39 – L2TRU truss element

As is possible to see in Figure 40 the FE model has different physical properties that correspond to the geometry and the characteristics at each level of the tower. In the structural modal analysis it was chosen, as the 3D beam model previously described, the characteristics of the materials (see Table 2 in chapter 3) described in the Italian report made by "*Tecnobrevetti s.r.l., 2008*". The materials defined in the model are represented in Figure 40b. Almost the entire tower is in masonry; tuff and bricks compose the lower part while the highest part is only with bricks. The physical properties of the stone material were chosen for the columns of the two belfries and the slab of the belfry in the upper part of the tower (see Figure 40b). The timber properties were used to define the roof of the tower.

The full model comprises 23,780 elements with 28,216 nodes. As boundary condition, the base section of the tower was pinned.

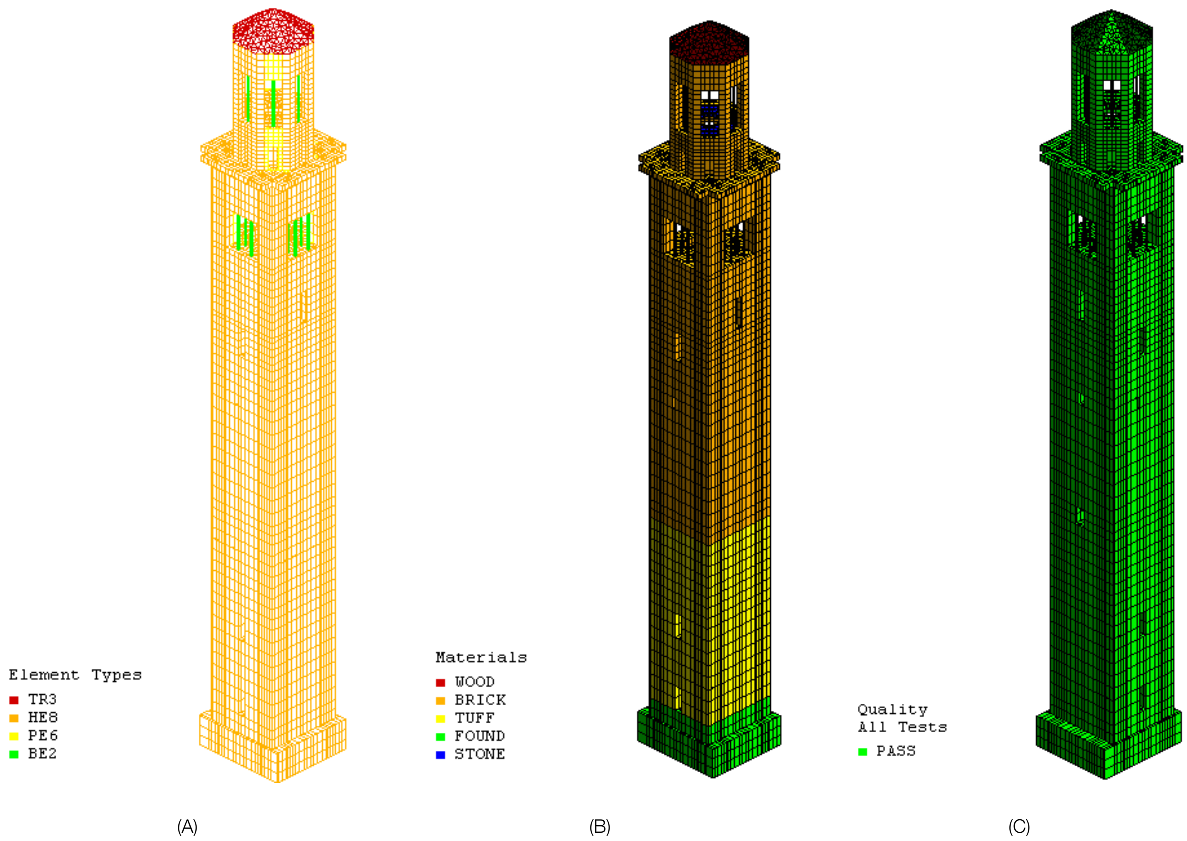


Figure 40 – Solid Model: a) Types of element that compose the FE model; b) Different materials of the tower c) Mesh quality test of the FE model

5.4 EIGENVALUE ANALYSIS OF THE SOLID MODEL

The mode shapes obtained from the eigenvalue analysis of the 3-D solid FE model follow also the behavior of the experimental results. Only the first five modes are presented in Figure 41 in order to compare them with the experimental results from the dynamic identification. The results of the frequencies are shown in Table 7.

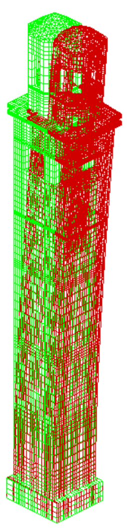
The numerical results of the frequencies of the 3D solid FE model show that there is a difference again in comparison with the experimental values. Considering the frequencies obtained from the Beam Model (Table 5) is possible to observe that the results from the two different models are very similar. This allows to state that the previous beam model is adequate and that simplifying the tower as a cantilever beam is adequate for its structural analysis, allowing namely a complex load representation, e.g. as a time history analysis. The results of the numerical analysis also indicate that the frequencies of the 3D model should be higher. To increase this value, several structural analyses were carried out and are described in the following Model Updating section.

EXPERIMENTAL FREQUENCIES			NUMERICAL FREQUENCIES SOLID MODEL	
Mode 1	0.65 Hz	Bending1	0.40 Hz	Bending1
Mode 2	0.67 Hz	Bending2	0.41 Hz	Bending2
Mode 3	2.48 Hz	2° ord. B1+B2	2.00 Hz	2° ord. B1+B2
Mode 4	2.55 Hz	2° ord. B1+B2	2.06 Hz	2° ord. B1+B2
Mode 5	3.03 Hz	Torsion	2.58 Hz	Torsion

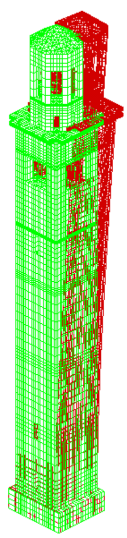
Table 7 – Comparison between the experimental frequencies and the 3D solid FE model frequencies

5.5 MODEL UPDATING FOR THE 3D FE SOLID MESH

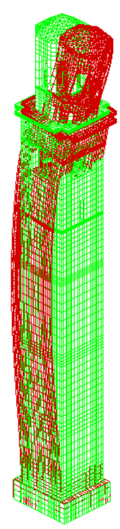
Modal updating is the process of adjusting an initial model, using data obtained from a vibration test, so that the model correctly represents the dynamic behavior of the structure. The adjective "correct" written in the previous sentence is a variable feature when applied to a mathematical model of a structure and can be defined and interpreted in different ways. The meaning accepted here is: capable to adequately representing the measured modal properties of the structure or the measured response functions of the structure. Normally there are two main strategies to update the model: by directly changing the individual elements in the mass and stiffness matrices of the model or by changing the values of physical/design parameters used to construct the individual component sub-matrices.



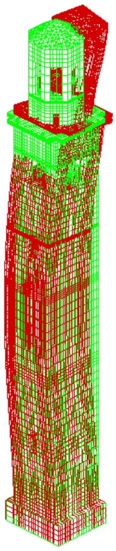
Mode Shape 1
Frequency= 0.398 Hz
Bending, direction (N-S, x-x)



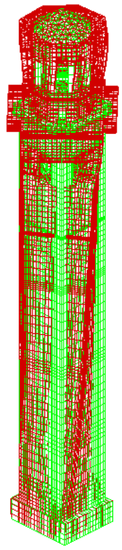
Mode Shape 2
Frequency= 0.411 Hz
Bending, direction (E-W, y-y)



Mode Shape 3
Frequency= 1.999 Hz
Bending, direction (N-S, x-x)



Mode Shape 4
Frequency= 2.059 Hz
Bending, direction (E-W, y-y)



Mode Shape 5
Frequency= 2.582 Hz
Torsion

Figure 41 – Modal shapes and frequencies of the SOLID FE model

Depending on the knowledge level, a numerical model consists of several assumptions, including geometric simplifications, estimation of mechanical properties, homogenization and inelastic behavior simplifications. Those features of modeling may cause improper definitions such as:

- Model Structure Errors: occurring when the physical definitions are uncertain or due to the inadequate consideration of inelastic phenomena;
- Model Parameter Errors: containing the application of inappropriate boundary conditions and any assumptions used in order to simplify the model
- Model Order Errors: arising in the discretization of complex systems, they are commonly faced when generating the meshes of models.

Inaccuracies and incompatibility are expected phenomena in the modeling of complex historic buildings. It is also noted that, in structural dynamics, experimental modal analysis is used for the determination of modal data (natural frequencies, mode shapes, generalized masses and damping factors). In model updating, usually batch-processing techniques are used to generate improved numerical models in order to obtain predictions for modified structural configurations. However, the mass, stiffness and damping parameters in the updated model should be physically meaningful (Mottershead & Friswell, 1993).

5.5.1 MODIFICATION OF THE ELASTIC MODULUS

From the structural eigenvalue analysis of the 3D FE solid model, it was observed that the adopted physical properties were not acceptable. The numerical values of the frequencies, which are much lower than the experimental results, indicate that the Young´s Modulus of the materials needs adjustment. The timber material of the roof and the stone material of the columns were not considered in the model updating because they are irrelevant in the general results.

In the different analyses, for each material, the Elastic Modulus was increased by 50% of its original valuel.

In the first analysis the stiffness of the foundation and the lower part of the tower, which is composed by tuff and bricks, was increased by 50%. Therefore for the foundation the Young Modulus changed from 8000 MPa to 12000 MPa and for the "tuff" part from 2700 MPa to 4050 MPa. Table 8 shows the results obtained.

NUMERICAL RESULTS OF THE ORIGINAL 3D MODEL			NUMERICAL RESULTS OF THE SOLID MODEL WITH FOUND/TUFF INCREASE	
Mode 1	0.398 Hz	Bending1	0467 Hz	Bending1
Mode 2	0.411 Hz	Bending2	0.481 Hz	Bending2
Mode 3	1.999 Hz	2° ord. B1+B2	2.126 Hz	2° ord. B1+B2
Mode 4	2.059 Hz	2° ord. B1+B2	2.186 Hz	2° ord. B1+B2
Mode 5	2.582 Hz	Torsion	2.866 Hz	Torsion

Table 8 – Comparison between the numerical frequencies of the original 3D solid model with the model with Etuff and Efound increased

In the second analysis the stiffness of the masonry part of the tower, which is composed only by bricks, was increased by 50%. Therefore the Young Modulus changed from 3400 MPa to 5100 MPa. Table 9 shows the results obtained.

A third analysis was carried out considering only the increase of the Young´s modulus of the foundation and of the "tuff" part. The results showed that the foundation does not affect significantly the frequencies but the "tuff" stiffness has a large improvement in the results.

In addition observing the previous frequencies and comparing the differences between the increase of the "brick" and the "foundation and tuff" part, in Tables 8 and 9, it is possible to understand that the stiffness of the lower part of the tower, made by tuff and bricks, affects more the values obtained by the numerical structural analysis.

NUMERICAL RESULTS OF THE ORIGINAL 3D MODEL			NUMERICAL RESULTS OF THE SOLID MODEL WITH BRICK INCREASE	
Mode 1	0.398 Hz	Bending1	0411 Hz	Bending1
Mode 2	0.411 Hz	Bending2	0.423 Hz	Bending2
Mode 3	1.999 Hz	2° ord. B1+B2	2.265 Hz	2° ord. B1+B2
Mode 4	2.059 Hz	2° ord. B1+B2	2.333 Hz	2° ord. B1+B2
Mode 5	2.582 Hz	Torsion	2.771 Hz	Torsion

Table 9 – Comparison between the numerical frequencies of the original 3D solid model with the models with Ebrick increased

After these considerations it was decided to increase the Young's Modulus of the lower part of the tower, made by tuff and bricks, to reach the same frequencies obtained by the experimental test. From the original value of 2700 MPa only with the value of 15000 MPa it is possible to obtain the same first frequency of the experimental result. Table 10 shows the frequencies of the first five modes.

EXPERIMENTAL FREQUENCIES			NUMERICAL FREQUENCIES OF THE SOLID MODEL WITH Etuff=15GPA	
Mode 1	0.6534 Hz	Bending1	0.6508 Hz	Bending1
Mode 2	0.6657 Hz	Bending2	0.6274 Hz	Bending2
Mode 3	2.483 Hz	2° ord. B1+B2	2.447 Hz	2° ord. B1+B2
Mode 4	2.551 Hz	2° ord. B1+B2	2.4995 Hz	2° ord. B1+B2
Mode 5	3.028 Hz	Torsion	3.226 Hz	Torsion

Table 10 – Comparison between the experimental frequencies and the 3D solid FE model frequencies with Etuff=15 GPa

It is interesting to observe that the results are really close with the experimental frequencies. In the Table 11 the ratio between the experimental values and the numerical results is calculated.

EXPERIMENTAL VALUES		ORIGINAL MODEL	RATIO	Etuff=15GPa	RATIO
Mode 1	0.6534 Hz	0.398 Hz	1.64	0.6508 Hz	1.00
Mode 2	0.6657 Hz	0.411 Hz	1.81	0.6274 Hz	0.99
Mode 3	2.483 Hz	1.999 Hz	1.24	2.447 Hz	1.01
Mode 4	2.551 Hz	2.059 Hz	1.24	2.4995 Hz	1.02
Mode 5	3.028 Hz	2.582 Hz	1.17	3.226 Hz	0.94

Table 11– Comparison between the experimental frequencies and the 3D solid FE model frequencies with Etuff=15 GPa with the calculation of the ratio

The ratio calculated in Table 11 shows that there is a significant difference between the results of the original model and the results with a stiffer material in the lower part of the tower. As the value found for the updated Young's modulus seems rather high, it is likely that the stiffening effect found for the lower part is due, to some extent, to the boundary conditions (the tower is connected to adjacent buildings). This is also addressed in the next section.

5.5.2 LATERAL SPRINGS

Around the four lateral walls of Lamberti Tower there are adjacent buildings up to 20.5 m. In the Figure 42 is possible to see the plan of the tower and in the Figure 43 is possible to see the section N-S of the tower with the lateral structures, where floors are connected to the tower.

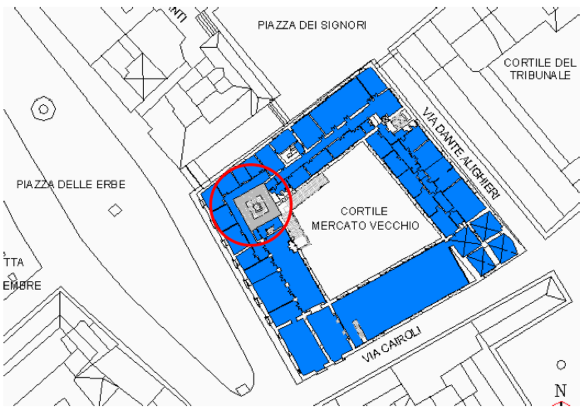


Figure 42 – Map of the Lamberti Tower, where is possible to see adjacent buildings.

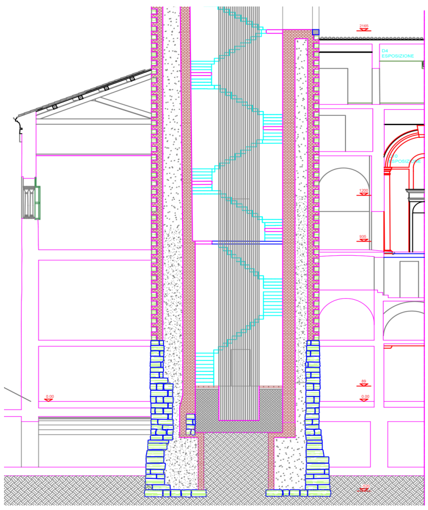


Figure 43 – Section of the Lamberti Tower

The presence of adjacent buildings, which are medieval masonry palaces, certainly influences the structural behavior of the tower. Some lateral springs were applied in the 3D FE solid model to simulate the impact of these constructions. It was chosen to insert some springs distributed up to the level of the slabs and arches. The stiffness of the springs was calculated considering the Elastic Modulus of the material of the adjacent building and the area of influence of each spring. The results are presented in Table 12.

From these results is possible to observe that considering lateral springs, the frequencies change significantly. Several analyses with different stiffness of the springs were carried out and it was curious to observe that by changing only the lateral springs stiffness it was difficult to reach the experimental values. Inserting lateral springs and increasing the Young's Modulus of the tuff part of the tower from the original value of 2700 MPa to 5600 MPa, it was possible to obtain the same first frequency of the dynamic identification test. The results, which seem more reasonable from the perspective of material data, are shown in Table 13.

EXPERIMENTAL FREQUENCIES			NUMERICAL FREQUENCIES OF THE SOLID MODEL WITH LATERAL SPRINGS	
Mode 1	0.6534 Hz	Bending1	0.531 Hz	Bending1
Mode 2	0.6657 Hz	Bending2	0.548 Hz	Bending2
Mode 3	2.483 Hz	2° ord. B1+B2	2.539 Hz	2° ord. B1+B2
Mode 4	2.551 Hz	2° ord. B1+B2	2.585 Hz	2° ord. B1+B2
Mode 5	3.028 Hz	Torsion	3.057 Hz	Torsion

Table 12 – Comparison between the experimental frequencies and the 3D solid FE model frequencies with lateral springs

EXPERIMENTAL VALUES		SOLID MODEL WITH SPRINGS AND TUFF INCREASE	RATIO
Mode 1	0.6534 Hz	0.6501 Hz	1.01
Mode 2	0.6657 Hz	0.6731 Hz	0.99
Mode 3	2.483 Hz	2.756 Hz	0.90
Mode 4	2.551 Hz	2.808 Hz	0.91
Mode 5	3.028 Hz	3.339 Hz	0.91

Table 13 – Comparison between the experimental frequencies and the 3D solid FE model frequencies with lateral springs and Etuff increased with the calculation of the ratio

The values obtained are again close to the experimental values. The ratio between experimental and numerical results shows that the first two frequencies are almost the same but from the third mode there is a small difference, probably because the effect of the springs is concentrated in some nodes and not distributed in the surface. Considering the previous comparison (Table 11), where it was considered a Young Modulus E=15000 MPa of the tuff material, it is possible to observe that the ratio is better and rather close to the experimental frequencies for all the 5 modes.

5.5.3 MODAL ASSURANCE CRITERION

The frequency response function matrix contains redundant information with respect to a modal vector and estimation of the modal vector for varying conditions or modal parameter estimation algorithms are valuable confidence factors for the evaluation of experimental modal vectors (Allemang, 2003).

The modal assurance criterion (MAC) provides a measure of consistency (linearity) between the estimated modal vectors. MAC is defined as:

$$\boldsymbol{MAC_{u,d}} = \frac{\left|\{\boldsymbol{\varphi_i^u}\}^{\boldsymbol{T}}\{\boldsymbol{\varphi_i^d}\}\right|^{\boldsymbol{2}}}{\{\boldsymbol{\varphi_i^u}\}^{\boldsymbol{T}}\{\boldsymbol{\varphi_i^u}\}\{\boldsymbol{\varphi_i^d}\}^{\boldsymbol{T}}\{\boldsymbol{\varphi_i^d}\}} \tag{22}$$

where φ^u and φ^d are the mode shape vectors of two different models. The MAC value will fall into a range 0 to 1, where the latter indicates 100% match of both modes shape vectors.

The modal assurance criterion is mostly used for validation of experimental results. In the same way it is practical to evaluate the correlation between experimental and numerical models. For this purpose it can give reasonable results for modal parameter estimation algorithms. In damage assessment and optimal sensor placement it can also be a useful tool.

Although MAC is judged as a useful tool for various cases, it should be stressed that in some cases the results may lead for wrong interpretations. In case that limited numbers of DOFs are measured and if the structure could not be observed properly, high MAC values cannot be interpreted as a good match. Still, this method is sensitive for high magnitudes. When higher magnitudes have a dominant effect on the results, erroneous points will have minor effects. The same problem may occur when the numbers of measurement points are not enough or well distributed. In order to have better results, DOFs in modal vector should be excited equally which requires the user to choose proper comparison points (Allemang J. R., 2003).

The MAC was calculated for the original 3D Solid Model, for the model with increase of Elastic Modulus of “tuff” material and for the model with lateral springs. The results are presented in Table 14.

ORIGINAL 3D MODEL			MODEL Increase 50% Etuff			MODEL Lateral Springs		
MODE SHAPE 1	MAC	0,9712	MODE SHAPE 1	MAC	0,9748	MODE SHAPE 1	MAC	0,9793
MODE SHAPE 2	MAC	0,9782	MODE SHAPE 2	MAC	0,9815	MODE SHAPE 2	MAC	0,9854
MODE SHAPE 3	MAC	0,5095	MODE SHAPE 3	MAC	0,5245	MODE SHAPE 3	MAC	0,5365
MODE SHAPE 4	MAC	0,5572	MODE SHAPE 4	MAC	0,5709	MODE SHAPE 4	MAC	0,5866
MODE SHAPE 5	MAC	0,0107	MODE SHAPE 5	MAC	0,012	MODE SHAPE 5	MAC	0,0142

Table 14 – Comparison between the MAC of the original model, model with Etuff increased and the model with springs.

As is possible to observe in Table 14 the MAC value of the first two mode shapes is very close to 1, for all the three analyses. The MAC for the mode 3 and 4 is not acceptable because usually the minimum value, to validate the correlation between experimental and numerical models, should be 0.7. The reason is probably because the experimental mode shapes are not parallel to one of the axis but rotated 45° (Figure 31). In fact, considering this angle in the calculation of the displacements, the MAC reaches reasonable values, as is possible to see in Table 15. It is noted that in a fully symmetric lumped model (e.g. if a beam model would be used) both sets of orthogonal eigenvalues are possible, given the point symmetry of the model.

This calls for better experimental readings and a more careful representation of the effects of the adjacent buildings. Mode 5, which is the torsion, was not considered in this last comparison because it is very sensitive to the modeling and to the measurements, and because at this mode the mass participation factor is not very high.

	ORIGINAL	MODEL WITH INCREASE of TUFF	MODEL WITH SPRINGS
Mode 1	0.9712	0.9748	0.9793
Mode 2	0.9782	0.9815	0.9854
Mode 3	0.8452	0.8643	0.8562
Mode 4	0.6821	0.8010	0.7812

Table 15 – Comparison between MAC values considering the deformed mode 3 and 4 of the tower inclined 45°

5.6 NON-LINEAR STATIC ANALYSIS (PUSHOVER)

The conventional pushover analysis is an incremental-iterative solution of the static equilibrium equations. The forcing function is a set of displacements or forces that are monotonically magnified during the analysis. During an increment the resistance of the structure is evaluated from the internal equilibrium conditions and the stiffness matrix is updated according to the iterative scheme adopted. The out-of-balance forces are iterated until a convergence criterion is satisfied. At convergence, the stiffness matrix is necessarily updated and another increment of displacements or forces is applied. The solution proceeds either until a predefined limit state is reached or the program fails to converge. It is presumed that the program has been sufficiently verified so that numerical, as opposed to structural, collapse is not operative. Three critical elements of the process are worthy of consideration, namely the forcing function nature, distribution and magnitude. In other words, should displacements or forces be applied, what is their adequate distribution (be it constant or variable) and what value of the applied action should be chosen at each load step, if they are not kept with a constant profile (A. S. Elnahai, 2001).

Therefore the pushover analysis can be defined in the following procedure:

- Appropriate lateral load patterns are applied to a numerical model of the structure and their amplitude is increased in a stepwise fashion;
- A non-linear static analysis is performed at each step, until the structure becomes unstable (and fails) or a specified limit consideration is attained;
- A pushover curve (or capacity curve) – usually base shear against top displacement – is plotted.

Static pushover analysis is an attempt by the structural engineering profession to evaluate the real strength of the structure and it is considered a useful and effective tool for performance based design.

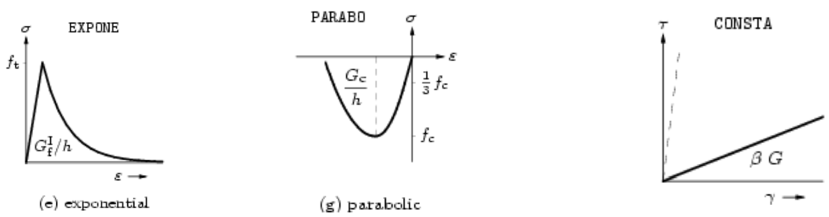
5.6.1 DEFINITION OF MASONRY CONSTITUTIVE LAW AND NON-LINEAR MATERIAL PROPERTIES

The non-linear behavior of the masonry of the tower was modeled by the adoption of a constitutive model based on total strain, called Total Strain Fixed Crack model. This model describes the tensile and compressive behavior of the masonry with one stress-strain relationship (DIANA, 2008). In this model the cracks are fixed according to the directions of the principal strains vector, and remain invariable during the loading process of the structure. According to (Mendes & Lourenço, 2008) the selection of the constitutive law should be done by making a balance between the accuracy, possibility to provide reasonable material parameters and computer cost of the calculation process. This model was selected because it provides good stability in the opening crack control and moderate computer cost.

Different strain-stress relationships were defined for tensile and compressive behavior of the material. It is known that the masonry has a linear hardening behavior up to its tensile strength, and the post peak tensile behavior is exponential softening (Roca & Lourenço, 2008-2009) (see Figure 44a). Regarding its compressive behavior, a model with parabolic hardening up to its compressive strength, was adopted followed by compressive post peak behavior idealized with a parabolic softening function (see Figure 44b). The post-cracked shear behavior was modeled using a retention factor of its linear behavior (see Figure 44c) which reduces its shear capacity according to:

$$\boldsymbol{G^{cr} = \beta \cdot G} \tag{23}$$

where $\boldsymbol{\beta}$ is the retention factor which varies between 1 and 0, and $\boldsymbol{G}$ is the shear modulus of the uncracked material.



(a) (b) (c)

Figure 44 – a) Tensile behavior b) Compressive behavior c) Shear behavior adopted for masonry (DIANA, 2008)

Finally, the crack bandwidth of each element was defined as a function of the area of each element:

$$\boldsymbol{h = \sqrt{A}} \tag{24}$$

where A is the area of each finite element.

The material properties defined in the structural analysis of the FE model are presented in Table 2. The values presented in this table, from a previous report (Tecnobrevetti, 2008) were not considered completely acceptable, after the eigenvalue analysis and the model updating, carried out and described in the previous sections of this thesis. Therefore, after defining the constitutive law of the material and its compressive and tensile behavior, non-linear properties of the material were defined following the recommendations shown by (Roca & Lourenço, 2012-2013). The masonry compressive strength was calculated according to the Equation 25 and the compressive fracture energy was calculated using the expression shown in Equation 26.

$$\boldsymbol{f_c = \frac{E}{500}} \tag{25}$$

where E is the elastic modulus of the material.

$$G_C = d \cdot f_c \tag{26}$$

where the d is the ductility index, equal to 1.6 mm.

The masonry tensile strength was adopted as a small fraction of the compressive strength, and the Tensile Fracture Energy used followed the recommendations of (Roca & Lourenço, 2012-2013).
The set of non-linear material properties used in the pushover are shown in Table 16.

:	Foundations with stone masonry	Masonry: course of bricks and tuff up to the level of 21 m	Masonry with only bricks above the level of 21 m	Small columns in compact stone
Compressive strength	f_k = 8 N/mm^2	f_k = 8 N/mm^2	f_k = 4 N/mm^2	f_k = 15 N/mm^2
Tensile strength	f_t = 0.3 N/mm^2	f_t = 0.4 N/mm^2	f_t = 0.3 N/mm^2	f_t = 0.5 N/mm^2
Modulus of elasticity	E = 8000 N/mm^2	E = 15000 N/mm^2	E = 3400 N/mm^2	E = 8000 N/mm^2
Tensile fracture energy	Gf = 0.05 N/mm^2	Gf = 0.05 N/mm^2	Gf = 0.03 N/mm^2	Gf = 0.02 N/mm^2
Compressive fracture energy	Gc=8 N/mm^2	Gc=10 N/mm^2	Gc=6 N/mm^2	Gc=6 N/mm^2
Coefficient of Poisson	v = 0.025	v = 0.00	v = 0.00	v = 0.025

Table 16 – Non-linear properties used in the pushover analysis

5.6.2 RESULTS

The non-linear static analyses were performed considering the 3D FE Beam Model and the 3D FE Solid Model. For the pushover analysis it was chosen the calibrated model with an increase of stiffness in the lower part of the tower (the model with Young Modulus of 15000 MPa described in the previous section), given their simplicity. A horizontal acceleration was applied by small increments so that the horizontal load acting on the elements was increased proportional to each element mass. In order to plot the capacity curve of the structure, the node that had the maximum displacement during the analysis was selected, which is the node located on the top of the tower. The Modified Newton-Raphson method was chosen for global convergence.

For the 3D beam model the resultant capacity curve for this analysis is plotted in Figure 45. In the curve the displacements, in meters, are represented in the x-axis while the load factor (the ratio between the resultant of the reactions and the mass) is the y-axis. In the plot, it is possible to observe that there is a clear initial linear behavior with a high slope and a maximum load coefficient of 0.11g is reached. After the displacement of 0.1 m, the structure assumes a non-linear behavior exhibited by the curve of the diagram. It is noted that the obtained displacements at the end of the analysis are very large and non-linear geometrical effects would play a role at this stage.

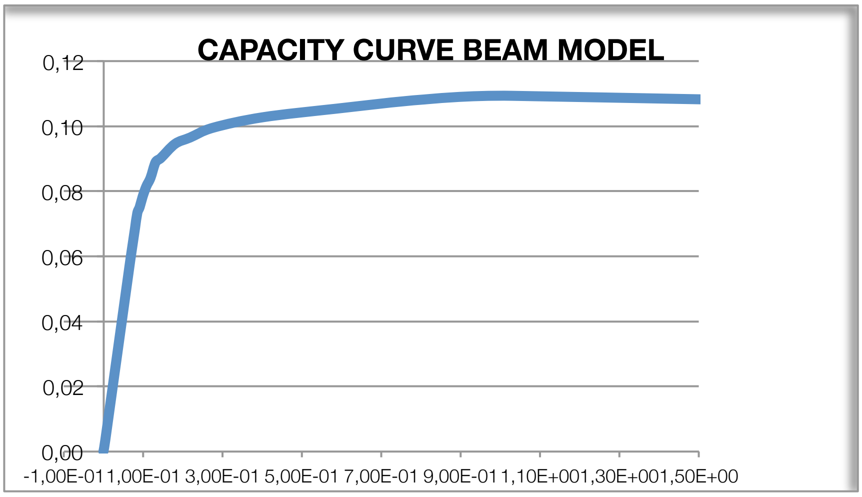


Figure 45 – Capacity curve of pushover analysis of the BEAM MODEL

The principal tensile strain distribution at the peak of the curve is shown in Figure 46a, where some cracks are clearly identified at the base of the tower. Figure 46b shows the strain distribution of the beam model at the end of the curve, when the tower reaches considerable displacements, where the failure mechanism with rotation at the base is clearly defined.

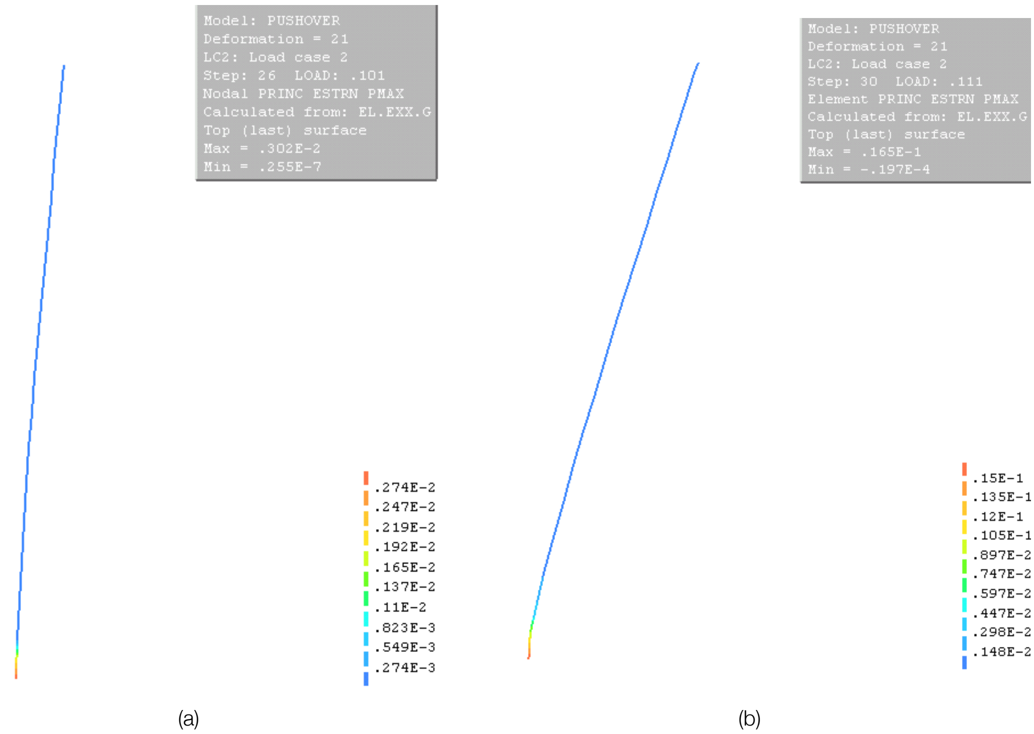


Figure 46 – Maximum principal tensile strains of the pushover analysis of the BEAM MODEL a) at the beginning of the nonlinear curve b) at the point of 1.0m

The resultant capacity curve of the 3D beam solid model for this analysis is plotted in Figure 47. In the plot, it is possible to observe that there is a clear initial linear behavior with a high slope and a maximum load coefficient of 0.105g is reached. After the displacement of 0.12 m, the structure assumes a non-linear behavior exhibited by the curve of the diagram. Again, it is noted that the obtained displacements at the end of the analysis are very large and non-linear geometrical effects would play a role at this stage.

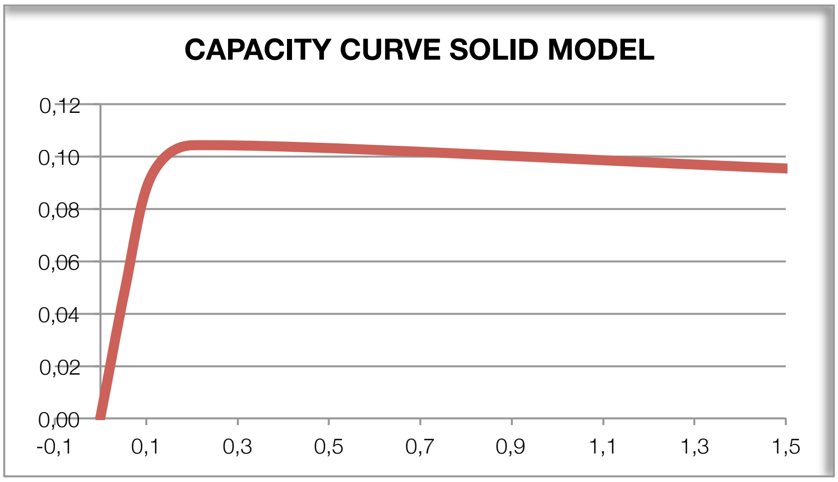


Figure 47 – Capacity curve of pushover analysis of the SOLID MODEL

The principal tensile strain distribution of the peak is shown in Figure 48, where some cracks are clearly identified at the base of the tower, between the foundation and the beginning of the masonry wall. Also some damages appear around the openings, in particular in the corners of the first belfry characterized by the first slab and a balcony.

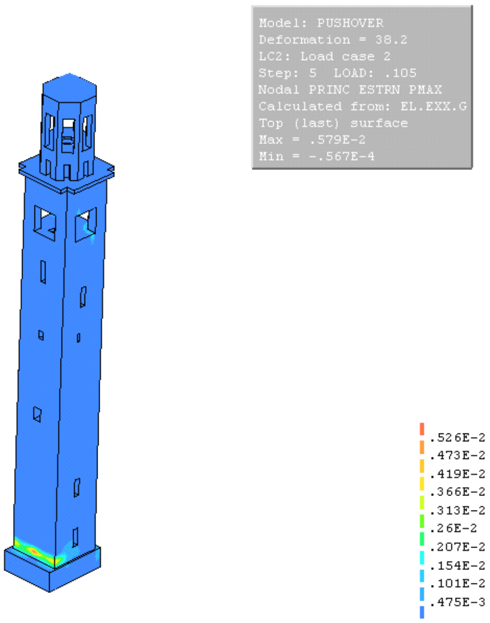


Figure 48 – Maximum principal strain distribution of the pushover analysis in the solid model at the pick of the curve

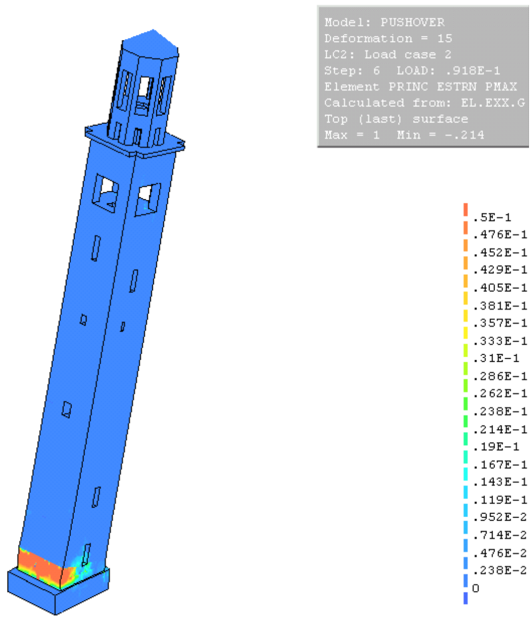


Figure 49 – Maximum principal strain distribution of the pushover analysis in the solid model at the end of the curve

In Figure 49 the strain distribution of the model is showed. At this step the displacement of the node on the top of the tower, considered in the pushover analysis, is 1.5 m from the initial position. This is a step where in the reality the masonry tower would already be collapsed.

It is interesting to compare the capacity curves of the beam model and the solid shown in Figure 50 the two plots are reasonably close, as the cracking load and the ultimate loads are similar. Some difference is present in terms of displacements and it is important to underline that the total mass of the two models is not the same, because many details were not considered in the definition of the beam model. For instance the timber roof, the openings, the slabs of the two belfry and the columns were modeled only in the 3D solid model. In the non-linear static analysis the weight of the structure is an important parameter that defines the load factor (base shear/self-weight). This could be an additional reason, besides the different model used, to justify why the two curves behave with some difference.
Still, comparing the two capacity curves, it is possible to observe that the first slope of the initial straight-line (i.e. the initial stiffness) is the same in the two models.

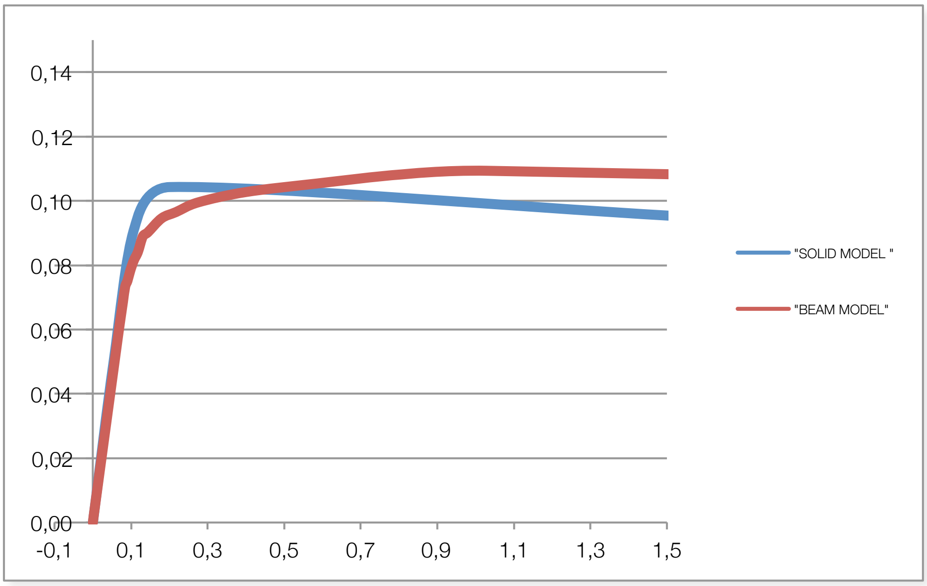


Figure 50 – Comparison between the capacity curve of the BEAM MODEL and of the SOLID MODEL

6. CONCLUSIONS AND RECOMMENDATIONS FOR EVENTUAL FUTURE STUDIES

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This work of thesis aimed at developing a finite element model of the medieval masonry tower “Torre dei Lamberti” (Italy). Two different models were made using 3D elements based on the Finite Element Method. The first model was made with 3D composite beams (Beam model), while the second model was made with 3D solid elements (3D model), being both models analyzed with the commercial finite element code DIANA.

During the construction of the model a dynamic identification on the Lamberti Tower was performed. It was clear that this process of experimental identification is a necessary step to develop a fully realistic and reliable model. In fact, thanks to these results a comparison between the numerical and experimental frequencies was possible.

The beam model and the solid model allowed to state that the mode shapes are similar to the configurations obtained experimentally but the frequencies obtained in the models were much different from the frequencies obtained experimentally. From the structural eigenvalue analysis of the 3D FE solid model, it was observed that the adopted physical properties or the boundary conditions were not acceptable. The numerical values of the frequencies, which are much lower than the experimental results, indicate that the Young´s Modulus of the materials and / or the boundary conditions needed adjustment. For this reason a model updating was carried out and the numerical model was modified by manual modification techniques. Firstly the Elastic Modulus of each masonry part of the tower was modified and also the boundary conditions of the tower were changed in order to consider the effects of adjacent buildings to the tower. The results of this comparison showed that, considering lateral springs, the frequencies change significantly. Several analyses with different stiffness of the springs were carried out and it was interesting to observe that by changing only the lateral springs stiffness it was difficult to reach the experimental values. Inserting lateral springs and increasing the Young's Modulus of the tuff part of the tower from the original value of 2700 MPa to 5600 MPa, it was possible to obtain the same first frequency of the dynamic identification test.

The modal assurance criterion (MAC) results reach reasonable values and showed that the numerical results are close to the experimental values.

The non linear static analysis carried out for the 3D solid model and for the 3D beam model allowed to say that a failure mechanism with rotation at the base of the tower, typical for tower structures, was clearly defined. The obtained displacements at the end of the analysis are very large and non-linear geometrical effects would play a significant role. Comparing the two capacity curves, of the beam and solid model, it was also possible to observe that the models are similar.

As a conclusion, it can be said that the models developed in this thesis could be improved with a deeper numerical calibration. Due to the lack of time, only an initial structural analysis was performed, using pushover analysis. In fact, this analysis could be improved considering the models updated with different boundary conditions. Simplified structural analyses with a kinematic approach and non-linear dynamic analyses could also be performed, starting from the models presented in this thesis, for a detailed safety assessment.

To improve the efficiency of the model and get more accurate information, accurate material parameters need to be found. Therefore NDT and MDT should be performed. A deeper investigation is recommended, especially for what concern the material parameters of the different type of masonry that compose the structure.

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